Accuracy Round Solutions

LMT Spring 2026

May 9, 2026

1. **[7]** Square $ABCD$ has side length 8. Sides $\overline{AB}, \overline{BC}$, and $\overline{CD}$ have midpoints P, Q, and R respectively. Segments $\overline{BD}$ and $\overline{QD}$ intersect segment $\overline{PR}$ at points X and Y respectively. Find the area of quadrilateral $BQYX$.

   Proposed by: Ben, Jerry, Calvin, William, Larry

   Solution. $\boxed{12}$

   $\overline{BQ}$ is half the length of side $\overline{BC}$ so it has length 4. $\overline{XY}$ is the midline of triangle $\triangle BQD$ so it has length 2. Quadrilateral $BQYX$ is a trapezoid with parallel bases $\overline{BQ}$ and $\overline{XY}$ and height 4. So, the area of the quadrilateral is $\frac{2+4}{2} \cdot 4 = \boxed{12}$ □

2. **[7]** Jerry has five distinct positive integers. Their mean is 10, their median is 11, and the largest number is 5 more than the second largest number. Find the largest possible value of Jerry's smallest number.

   Proposed by: Isabella Li

   Solution. $\boxed{4}$

   Let the five integers in the problem be $a < b < c < d < e$, where

   $$c = 11, \quad e = d + 5, \quad a + b + c + d + e = 50.$$

   This simplifies to $a + b + 2d = 34$. To maximize a, we let d be as small as possible, so $d = 12$ and $a + b = 10$, which gives $a = \boxed{4}$ when plugged into $a + b + c + d + e = 50$. □

3. **[8]** Peter and Larry both pick a positive integer from 1 to 6, inclusive, and secretly tell their friend Tnag their respective numbers. Then, the following four true statements are made:

   - Tnag announces that "Peter's number is at least 3 times as large as Larry's number".
   - Larry responds, "I still don't know Peter's number".
   - Peter says, "And I don't know Larry's".
   - Finally, Larry states, "Oh, now I know Peter's number".

   Find the ordered pair (Larry's number, Peter's number).

   Proposed by: Ryan Tang

   Solution. $\boxed{\text{Voided}}$

   The first statement implies only

   $$(1,4),(1,5),(1,6),(2,6)$$

   are possible. The second implies that $(1,6),(2,6)$ are possible. The third implies $(1,6)$ is the solution. □

4. **[8]** Find the number of real numbers x that satisfy the equation

   $$\lceil x \rceil + \lfloor x \rfloor = \frac{4049}{2026} x.$$

   Proposed by: Larry Cui, Ryan Tang

Solution. $\boxed{1351}$

Clearly, the only integer solution is $x = 0$. Suppose $\lfloor x \rfloor = k$ and $x = k + r$. Thus, $2k + 1 = \frac{4049}{2026}k + \frac{4049}{2026}r$. Thus,

$$k = \left(\frac{4049}{2026}r - 1\right)\frac{2026}{3} = \frac{1}{3}(4049r - 2026).$$

In particular, note that we need $r = \frac{a}{4049}$ and $4049r - 2026 \equiv 0 \pmod 3$. This is equivalent to $a \equiv 1 \pmod 3$. Thus, $a \in \{1, 4, \ldots, 4048\}$ which gives 1350 solutions. Thus, the answer is 1351. □

5. **[11]** Let $N = 76393728$ be an 8-digit number that is divisible by 128. Find the number of 8-digit multiples of 128 M such that M and N differ by one digit. *For example,* 76793728 *is a valid M since only one of its digits differ from N's digits and it is divisible by 128* .

Proposed by: Ryan Tang

Solution. $\boxed{14}$

We can change the first digit by any amount since 10^7 is divisible by 128. Thus, there are 8 ways for this. To change the second digit, it must be by a multiple of 2. Thus, $0, 2, 4, 8$ are valid, which is 4. To change the 3rd digit, it must be a multiple of 4. Thus, only 7 is valid. Finally, to change the 4th digit, it must be by a multiple of 8, which works. To change everything else requires an addition/subtraction of a number > 10 so we are done. Thus, the answer is $8 + 4 + 1 + 1 = \boxed{14}$. □

6. **[12]** A bag starts with 2 red balls and 1 green ball. At each step, two balls are drawn uniformly at random without replacement from the bag, then returned to the bag, and 1 green ball is added. The process ends when both drawn balls are red. Find the probability that the process never ends.

Proposed by: James Wu

Solution. $\boxed{\frac{1}{3}}$

Given that the process survived through step $n - 1$, the probability it ends at step n is

$$\frac{1}{\binom{n+2}{2}} = \frac{2}{(n+1)(n+2)}$$

So the probability of survival is

$$1 - \frac{2}{(n+1)(n+2)} = \frac{n^2 + 3n}{(n+1)(n+2)} = \frac{n(n+3)}{(n+1)(n+2)}$$

The probability that the process doesn't end by step n is therefore

$$\prod_{i=1}^{n} \frac{i(i+3)}{(i+1)(i+2)} = \prod_{i=1}^{n} \frac{i}{i+1} \cdot \prod_{i=1}^{n} \frac{i+3}{i+2} = \frac{1}{n+1} \cdot \frac{n+3}{3} = \frac{n+3}{3(n+1)}$$

The problem asks for the value as n approaches infinity, which is $\boxed{\frac{1}{3}}$. □

7. **[12]** Find the unique 4-digit positive integer $\underline{abcd}$ such that $a \neq 0$, $c \neq 0$, and

$$\underline{abcd} = 2\left(\underline{cdab}\right) + 2.$$

Note: $\underline{abcd}$ refers to the 4-digit number formed by writing out the digits a, b, c, and d in that order.

Proposed by: Peter Bai

Solution. $\boxed{6532}$

Since digits a and b are always placed together, it is useful to treat them as a single unit. Let $x = \underline{ab}$ and $y = \underline{cd}$ be two-digit positive integers. The given condition then becomes

$$100x + y = 2(100y + x) + 2 \implies 98x - 199y = 2.$$

Taking this equation modulo 98, we get

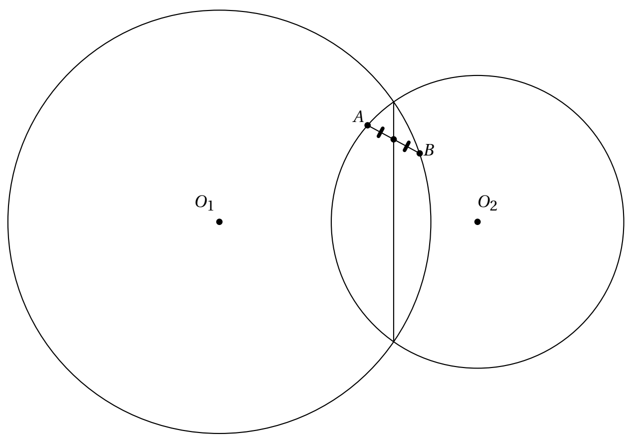
$$y \equiv 32 \pmod{98}.$$

Thus, since $y < 100$, we have $y = 32$. Plugging this back in,

$$98x = 199y + 2 = 199 \cdot 32 + 2 = 98 \cdot 64 + 96 + 2 = 98 \cdot 65 \implies x = 65.$$

□

8. **[15]** In the diagram below, the two marked segments have the same length. Furthermore, the distance from O_1 and O_2 to line AB are 6 and 5, respectively, and the radius of the circle centered at O_1 is 12. Find the radius of the circle centered at O_2. *Note: Diagram is not necessarily drawn to scale.*



Proposed by: Ryan Tang

Solution. $\boxed{\sqrt{133}}$

Let X be the midpoint of AB. Extend AB to intersect ω_1, ω_2 again at C, D, respectively. Let M, N be the midpoints of AC and BD, respectively. Note that $O_1M = 6, O_2N = 5$. Due to Power of a point, $XA \cdot XC = XB \cdot XD$, but we also know $XA = XB$, so $XC = XD$. Thus, $AC = BD$. Thus, $AM = BN$, but then $r_1^2 - d_1^2 = r_2^2 - d_2^2$. Plugging in the values, we get $\boxed{\sqrt{133}}$. □

9. **[16]** A 2026×2026 grid is filled with integers from 0 to 2026, inclusive, such that the same number does not appear twice in any row or column. Call a cell of a grid *complete* if every integer from 0 to 2026 either appears in its row or in its column. Find the largest possible fraction of cells that are *complete*.

Proposed by: Henry Eriksson

Solution. $\boxed{\frac{2025}{2026}}$

We will start with an upper bound via the pigeonhole principle. In a fixed column, exactly one number will be missing. Without loss of generality, take this number to be 0. For all of the cells in our column, it is complete if and only if there is a 0 in its row. There are 2025 other columns, so there can only be up to 2025 zeros, so only up to 2025 of the 2026 cells in the column can be complete. Doing this over all of the rows, we find that there cannot be more than $2025 \cdot 2026$ complete cells.

Now we will construct a grid with $2025 \cdot 2026$ complete cells. Make the top row $1, 2, \ldots, 2026$. Then define each cell to be one more than the cell above it, going back to zero after passing 2026. Then the i'th column and the i'th row are both missing the number $i-1$. This means that any cell on the main diagonal is missing the same entry in its row and in its column, and thus is incomplete, but any cell off of the main diagonal has a different entry missing in its row and column, and therefore is complete. This means that this construction gives us our upper bound of $2025 \cdot 2026$ complete cells.

The question asks for the maximal proportion, so our final answer is $\frac{2025\cdot 2026}{2026^2} = \boxed{\frac{2025}{2026}}$. □

10. **[17]** Define S_n as the set $\left\{\left\lfloor \frac{n}{1} \right\rfloor, \left\lfloor \frac{n}{2} \right\rfloor, \cdots, \left\lfloor \frac{n}{n} \right\rfloor\right\}$. For example, $S_{15} = \{1,2,3,5,7,15\}$. Call a positive integer n *golden* if $9 \in S_{n-1}$ but $9 \notin S_n$. Find the sum of all n that are *golden*.

Proposed by: Ryan Tang

Solution. $\boxed{360}$

We solve the general case for $k = 9$.

Suppose that $\left\lfloor \frac{n-1}{\ell} \right\rfloor = k$. Observe that $\frac{n}{\ell} - \frac{n-1}{\ell} = \frac{1}{\ell}$ so

$$\left\lfloor \frac{n}{\ell} \right\rfloor - \left\lfloor \frac{n-1}{\ell} \right\rfloor \le 1,$$

so $\left\lfloor \frac{n}{\ell} \right\rfloor = k+1$. It can be seen that this implies that $n = (k+1)\cdot \ell$. This directly implies that $\left\lfloor \frac{n-1}{x} \right\rfloor = k$ has exactly one solution in x (it is $x = \ell$). Thus,

$$\left\lfloor \frac{n-1}{\ell+1} \right\rfloor < k, \left\lfloor \frac{n-1}{\ell-1} \right\rfloor \ge k+1.$$

These are equivalent to

$$(\ell-1)(k+1) \le n-1 < (\ell+1)k,$$

which when plugging in $n = (k+1)\ell$ implies that $\ell < k+1$. We also require $\left\lfloor \frac{n}{\ell+1} \right\rfloor < k$, or $n < (\ell+1)k$. Plugging in $n = (k+1)\ell$, we get $\ell < k$.

Thus, the set of n such that k is golden is $(k+1)\ell$ for $\ell < k$. Thus, the answer is

$$\sum_{\ell=1}^{k-1} (k+1)\ell = \frac{(k+1)k(k-1)}{2} = \boxed{360}.$$

□

11. **[18]** From 2026 equally spaced points on the circumference of a circle, four points are chosen uniformly at random and labeled A, B, C, and D, in clockwise order. Find the probability that the shorter arc between A and C has the same length as the shorter arc between B and D.

Proposed by: James Wu

Solution. $\boxed{\frac{1}{675}}$

Let the clockwise gaps be

$$AB = x_1, \quad BC = x_2, \quad CD = x_3, \quad DA = x_4,$$

where x_1, x_2, x_3, and x_4 are positive integers satisfying

$$x_1 + x_2 + x_3 + x_4 = 2026.$$

The minor arc between A and C is $\min(x_1 + x_2, 2026 - x_1 - x_2)$, and the minor arc between B and D is $\min(x_2 + x_3, 2026 - x_2 - x_3)$. For the lengths to be equal, they must either satisfy

$$x_1 + x_2 = x_2 + x_3 \iff x_1 = x_3$$

or

$$x_1 + x_2 + x_2 + x_3 = 2026 \iff x_2 = x_4.$$

Thus, we want the probability that at least one of $x_1 = x_3$ and $x_2 = x_4$ holds. The total number of positive solutions of x_1, x_2, x_3, x_4 is

$$\binom{2025}{3}.$$

We can use inclusion-exclusion:

- $x_1 = x_3$: we need
$$2x_1 + x_2 + x_4 = 2026.$$
Since there are $2025 - 2x_1$ choices of x_2 and x_4 for each $x_1 = 1, 2, \dots, 1012$, our sum is
$$\sum_{i=1}^{1012} 2025 - 2i = 1012^2.$$
- $x_2 = x_4$: the same as the previous case by symmetry.
- $x_1 = x_3$ and $x_2 = x_4$: we need
$$2a + 2b = 2026 \Longleftrightarrow a + b = 1013$$
The number of positive integer solutions is 1012.

Therefore, the probability is
$$\frac{2 \cdot 1012^2 - 1012}{\binom{2025}{3}} = \frac{1012 \cdot 2023 \cdot 6}{2025 \cdot 2024 \cdot 2023} = \boxed{\frac{1}{675}}$$

□

12. **[19]** Let $ABCD$ be a convex quadrilateral inscribed in a circle. Points X and Y are the feet from A and D to line BC. Suppose that the midpoint of BC is also the midpoint of XY, $AB = 25$, $CD = 39$, and $AD = 65$. Find BC.

Proposed by: Ryan Tang

Solution. $\boxed{33}$

The main claim is that AD is the diameter.

Proof: Let O be the midpoint of AD and let M be the foot from BC. Since $AX \parallel DY \parallel OM$, and O is the midpoint of AD, it follows that M is the midpoint of XY. Using the equal lengths condition, it follows that M is the midpoint of BC. Thus, O lies on the perpendicular bisector of BC. Furthermore, since $AB \neq CD$, we have AD and BC are not parallel, so the perpendicular bisector of AD is distinct from the perpendicular bisector of BC. Since O lies on both lines, O is the circumcenter and we are done.

To finish, by the Pythagorean theorem, $BD = 60$ and $AC = 52$. By Ptolemy's theorem, we get the answer
$$\frac{60 \cdot 52 - 25 \cdot 39}{65} = \boxed{33}.$$

□

13. **[TIEBREAKER]** Eddie picks a point P uniformly at random from the interior of a square S with side length 2, and draws a unit circle Γ centered at P. Estimate the expected value of the area of the region(s) consisting of points inside of S but outside of Γ. Express your answer in the form $a.bcdef$.

Proposed by: Peter Bai

Solution. $\boxed{2.06674068}$

We will estimate the area of the part of Γ that is inside of S, and then remember to subtract our result from 4 at the end to find our answer. If P is a vertex of S, then a quarter of Γ lies inside of S, with area
$$\frac{1}{4} \cdot \pi(1)^2 = \frac{\pi}{4}.$$
If P is the center of S, then the entirety of Γ lies inside of S, with area
$$\pi(1)^2 = \pi.$$
We can average these two extremes to get an estimate of the expected area in both Γ and S. This approach of linearly interpolating between a vertex and the center of S has two main sources of error:

- Although the midpoint of a side of S is "halfway" between a vertex and the center, the area in both S and Γ if P were to be located there would be $\pi/2$. Our approximation provides an overestimate of $5\pi/8$ in this region, which is the average of $\pi/4$ and π.

- For P close to the center of S, the area in both S and Γ is relatively large. This is because moving P slightly away from the center of S decreases this area by a comparatively small amount, as only a tiny slice of the edge of Γ gets moved outside of S. As a result, our approximation underestimates the area in this region.

Somewhat coincidentally, these details largely cancel out, and our approximation of

$$4 - \frac{\frac{\pi}{4} + \pi}{2} = 4 - \frac{5\pi}{8} \approx \boxed{2.0365}$$

is surprisingly close to the true value. □