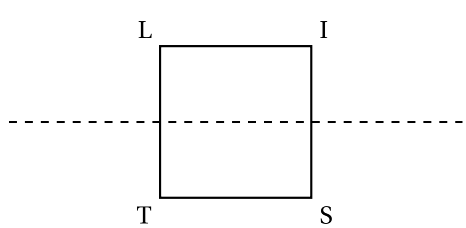
Speed Round

LMT Spring 2025

May 3, 2025

1. **[6]** If $2025x + 5202 = 2025 + 5202x$, find x.
2. **[6]** A triangle with integer side lengths has perimeter 30, and one side of length 10. Find the maximum possible length of the longest side of this triangle.
3. **[6]** Two standard 4-sided dice are rolled. Find the probability their product of the numbers rolled is a square number.
4. **[6]** A three digit number is formed with digits $1, 2, 3$, each used once, so that the first two digits form a prime number and the last two digits form a prime number. Find this three digit number.
5. **[6]** Topher is eating pickles. He eats a pickle at 12:00 AM midnight on Monday, and from then on, he eats a pickle every 45 minutes until he eats his last pickle on Friday at 4:30 PM. Find the number of times Topher eats a pickle.
6. **[6]** Jon is traveling 60 miles from Lexington High School to Lake Orz. If he travels at a rate of 15 mph for the first 30 miles, and 10 mph for the last 30 miles, find his average speed throughout the entire trip, in mph.
7. **[6]** Little John's right triangular room has two sides with length 20 and 25. Find the length of the last side if the area of his room is 150 square units.
8. **[6]** Equilateral triangle ABC has side length $AB = 1$. Point D lies on line BC such that $BD = 2025$ and C lies inside segment BD. Find the area of triangle ACD.
9. **[6]** Leo writes the number 1 on the board. Every minute, he either doubles the number on the board or adds 300 to it. Find the smallest possible number he can achieve after 15 minutes.
10. **[6]** Triangle ABC has area 1 with $AB = AC$. Let D and E be the midpoints of AB and AC. EB and DC intersect at F. Find the area of DEF.
11. **[6]** Find the number of 4-digit numbers that have at least 1 odd digit adjacent to an even digit.
12. **[6]** Let $P(n)$ denote the product of digits of n. Find the sum of $P(n)$ for all positive two-digit integers n.
13. **[6]** Square $LIST$ has side length 6. A line is drawn through the midpoints of LT and IS. Points R and H lie on this line and point O lies on side LI such that the areas of $SHORT$ and $LIST$ are equal. Find the length of RH.

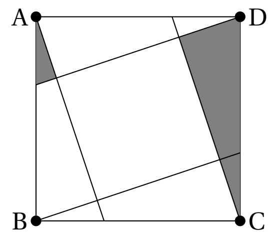


14. **[6]** Carter starts at $(0, 24)$ and heads towards $(24, 6)$ in a straight line. Halfway there, however, he remembers to get milk and turns 90 degrees clockwise and moves forwards in a straight line until he reaches the y-axis. After he gets the milk, he walks to $(24, 6)$ again in a straight line. Find the total distance he traveled.
15. **[6]** Find the sum of all distinct prime factors of 12345654321.

16. **[6]** Find the surface area that is enclosed on a sphere of radius 10 by three arcs of length 5π which follow the curvature of the sphere.

17. **[6]** Find the smallest integer $x \geq 2$ such that $3^x > x^9$.

18. **[6]** In the square below the area of the small triangle in the top left corner is 1, and the area of the larger triangle in the bottom right corner is 9. The shape in the middle is a square. Find the area of the large square.



19. **[6]** A 2-digit base 10 positive integer $\overline{XY}$ is called *esab* if there exists a positive integer $b < 10$ such that the base b representation of $\overline{XY}$ is $\overline{YX}_b$. Find the sum of all 2-digit esab numbers. Express your answer in base 10.

20. **[6]** For even n, define $f(n)$ to be the smallest odd prime p such that $n - p$ is not prime. Find the maximum value of $f(n)$ for all positive even $n > 10$.

21. **[6]** Every cell of a 3×3 grid can be colored black or white. For each black cell in the grid, a point is added for every adjacent black cell. Find the sum of points across all 2^9 possible grid colorings.

22. **[6]** Find the number of ordered 2025-tuples $(a_1, a_2, \ldots a_{2025})$ of non-negative integers satisfying

$$\left\lfloor \frac{a_1}{1} \right\rfloor + \left\lfloor \frac{a_2}{2} \right\rfloor + \cdots + \left\lfloor \frac{a_{2025}}{2025} \right\rfloor = 2025.$$

23. **[6]** Let ABC be an equilateral triangle and $\mathscr{P}$ a plane which does not intersect ABC. Let the projections of A, B, C onto $\mathscr{P}$ be A', B', and C'. Suppose $\angle B'A'C' = 90^\circ$ and A, B, C have heights 37, 23, and 16 to $\mathscr{P}$, respectively. Find AB^2.

24. **[6]** There exists some positive integer k such that the coefficient of x^k in the expansion of $(3x+1)^{2025}$ is maximized. Find k.

25. **[6]** Let $p = 2027$ be a prime and $f(x) = x^2 - 2$. Find the remainder when

$$\underbrace{f\left(f\left(\cdots f\left(\frac{p+5}{2}\right)\cdots\right)\right)}_{\text{2025 times}}$$

is divided by p.