LMT Spring 2025 Guts Round - Part 1

Team Name: ____________________

____________ 1. **[9]** The Einstein-Pythagoras equation states that

$$E = m\left(a^2 + b^2\right).$$

If $m = \frac{1}{17}$, $a = 16$ and $b = 30$, find the value of E.

____________ 2. **[9]** Find the minimum possible value that can be created using the numbers $2, 0, 2$, and 5, given that only the operations addition, subtraction, multiplication, and division can be used (no concatenation).

____________ 3. **[9]** Find the units digit of $2^{2025} + 0^{2025} + 2^{2025} + 5^{2025}$.

LMT Spring 2025 Guts Round - Part 2

Team Name: ____________________

____________ 4. **[10]** Drew has ten different paint colors that are split into three categories: There are three warm colors, three cool colors, and four neutral colors. Drew wants to mix two colors that are not from the same category. Find the number of ways he can choose these two colors.

____________ 5. **[10]** Apollo rolls a standard 2-sided die, 4-sided die, and 6-sided die. Find the probability he rolls three distinct numbers.

____________ 6. **[10]** Let A, B, C, D, P be points in the plane satisfying $PA = 5$, $PB = 6$, $PC = 7$, $PD = 9$. Find the maximum possible area of the quadrilateral formed by the points A, B, C, D.

LMT Spring 2025 Guts Round - Part 3

Team Name: ____________________

____________ 7. **[11]** Find the sum of the positive factors of 2025 that have exactly 3 positive factors.

____________ 8. **[11]** There exist two positive integers a and b such that $\frac{22!\cdot 23!}{a!\cdot 24!} = \binom{b}{4}$. Find $a + b$.

____________ 9. **[11]** Define a sequence of numbers $n_1, n_2, \ldots$ such that $n_i n_{i+1} = n_{i+2}$ for all positive integers i. Given that $n_1 = n_{2026}$ and $n_2 = n_{2027}$, find the number of possible ordered tuples $(n_1, n_2 \ldots n_{2024}, n_{2025})$.

LMT Spring 2025 Guts Round - Part 4

Team Name: ____________________

____________ 10. **[12]** Find

$$\sum_{a\geq 0}\sum_{b\geq 0}\frac{1}{2^a 3^b 5^{a+b}}.$$

____________ 11. **[12]** Suppose a, b, c are the roots of $x^3 - ax^2 - bx - c$. Find all possible values of c.

____________ 12. **[12]** Call a number n *factormaxxing* if exactly 4 of its factors are in the set {2, 3, 4, 5, 8, 9, 16, 25, 27, 32, 81, 125, 243, 625, 3125}. Given that n has no prime factors other than $2, 3$, and 5, find the number of possible values of n that are *factormaxxing*.

LMT Spring 2025 Guts Round - Part 5

Team Name: ____________________

____________ 13. **[13]** There are 46 freshmen and 35 sophomores who place themselves in distinct cells of a 9×9 grid. In a move, two people can switch places (these people don't need to be adjacent). Sophomores all hate each other, so they don't want to be in adjacent cells. Find the smallest k for which we can always perform at most k moves to make sure no two sophomores are adjacent.

____________ 14. **[13]** A function $g(t) = 2t^3+9$ satisfies $g(x)-g(y) = 126$ for some positive numbers x and y. If $x-y=3$, find the value of $x+y$.

____________ 15. **[13]** Four spheres of radius 6 are mutually externally tangent. Point P is chosen such that it is d units from the center of each sphere. Find d.

LMT Spring 2025 Guts Round - Part 6

Team Name: ____________________

____________ 16. **[14]** Tiger is taking Day 1 of the 2025 IMO TST. The test consists of three problems. Tiger's solution to each problem obtains a score that is chosen uniformly at random from $[0,1]$. However, Tiger accidentally misorders his solutions! For problem 1, he submitted the solution that got the highest score, for problem 2, he submitted the solution that got the second highest score, and for problem 3, he submitted the solution that got the lowest score. Any solution corresponding to the right problem earns a score equal to the solution's score, while any problem in the wrong position gets a score of 0. Find the expected value of Tiger's score.

____________ 17. **[14]** Each second, ST rolls a fair 6-sided die and records the result. If the sum of the numbers ST has written ever exceeds 9, he will erase the last number he wrote. Find the expected number of times he needs to roll the dice before the sum is exactly 9.

____________ 18. **[14]** Semicircle O has diameter $AB = 12$. Arc $BC = 135^\circ$. Let D be the midpoint of arc $\widehat{BC}$. Find the area of the region bounded by the lines AD, CD and arc $\widehat{AC}$.

LMT Spring 2025 Guts Round - Part 7

Team Name: ____________________

____________ 19. **[15]** Let point P lie inside isosceles right triangle ABC with $AB = BC$. Suppose P has distances 3, 4, and 5 to sides AB, BC, and CA, respectively. Find AB.

____________ 20. **[15]** Find the number of nonnegative integers (a,b,c,d) such that

$$a+b\le 4,$$
$$a+d\le 4,$$
$$c+b\le 4,$$
$$c+d\le 4.$$

____________ 21. **[15]** Find the number of ordered quadruples of nonnegative integers w,x,y,z such that

$$\left\lfloor \frac{2025}{10^k}\right\rfloor = \left\lfloor \frac{w}{10^k}\right\rfloor + \left\lfloor \frac{x}{10^k}\right\rfloor + \left\lfloor \frac{y}{10^k}\right\rfloor + \left\lfloor \frac{z}{10^k}\right\rfloor$$

holds for all nonnegative integers k.

LMT Spring 2025 Guts Round - Part 8

Team Name: ______________________

______ 22. **[17]** Find the number of three digit perfect squares $\underline{a}\,\underline{b}\,\underline{c}$ for which $a(2025^2)+b(2025)+c$ is also a perfect square.

______ 23. **[17]** Find the number of positive integers c less than 2025 such that there exist positive integers a and b satisfying

$$2^a+2^b=c^2.$$

______ 24. **[17]** Sam and Jonathan take turns flipping a fair coin, and they stop when they collectively flip three heads in a row. Given Sam flips first, find the probability he flips the last head.

LMT Spring 2025 Guts Round - Part 9

Team Name: ______________________

______ 25. **[20]** Let ABC be an equilateral triangle with side length 1. Let Ω be the circumcircle of triangle ABC, and let P lie on minor arc AB of Ω. The tangent line at P intersects AB at D and AC at E such that $PD=PE$. Find the area of ADE.

______ 26. **[20]** Let a pair of integers (x,y) be called *yearly* if

$$(x+y)^2=100x+y.$$

For example, $(20,25)$ is a yearly pair. There exists two yearly pairs $(x_1,y),(x_2,y)$ such that $y<-100$ is maximized. Given that $x_2>x_1$, find (x_2,y).

______ 27. **[20]** Let $f(x)$ be a quartic polynomial with roots w,x,y,z such that

$$w=\frac{1}{5-xyz},\qquad x=\frac{1}{7-wyz},\qquad y=\frac{1}{9-wxz},\qquad z=\frac{1}{11-wxy}$$

and $f(0)=3$. Find $f(1+wxyz)$.

LMT Spring 2025 Guts Round - Part 10

Team Name: ______________________

______ 28. **[23]** Let $ABCD$ be a parallelogram with $AB=20$, $BC=25$. Let E be the midpoint of BC, and let F be the foot of the altitude from D to AE. Given that $AF=15$, find the area of triangle CDF.

______ 29. **[23]** Tiger is taking Day 2 of the 2025 IMO TST. The test consists of three problems. This time, Tiger decides to not look at the problem numbers. When doing the nth problem, he gives the problem a difficulty rating, chosen uniformly at random from $[0,n]$. He then orders the problems in increasing order of difficulty. Find the probability that none of the problems in Tiger's ordering are in the same place they were in for the original ordering.

______ 30. **[23]** Let ABC be a triangle such that $AB=3$, $AC=4$, $BC=5$. A circle through A intersects AB at P, AC at Q, and BC at R and S such that $\angle PQS=\angle QPR=60^\circ$. Find $\frac{BR}{CR}+\frac{BS}{CS}$.

LMT Spring 2025 Guts Round - Part 11

Team Name: ______________________

______ 31. **[26]** Two 10 by 10 grids with squares colored red and blue are called *evil* if for every row, column, or diagonal, the number of red cells is the same for both. Find the maximum number of red cells in common between two distinct 10 by 10 grids that are evil.

______ 32. **[26]** Let $ABCD$ be a cyclic quadrilateral. Given that $AB = CD = 2$ and the distance between any two points in the set $\{A, B, C, D\}$ is a positive integer, find the sum of all possible perimeters of $ABCD$.

______ 33. **[26]** Let $S = \{1,2,3,4\}$. Compute the number of functions $f : S \to S$ such that no $x \in S$ satisfy $f(f(x)) + x = 2f(x)$.

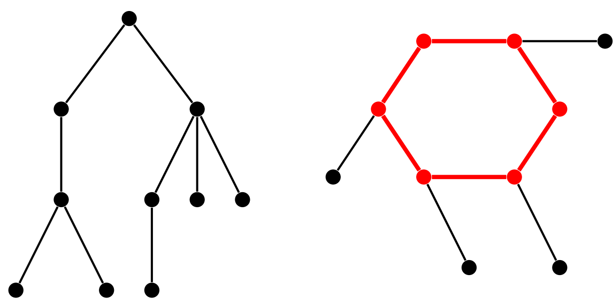
LMT Spring 2025 Guts Round - Part 12

Team Name: ______________________

______ 34. **[30]** Let G be a tree chosen uniformly at random from the set of unlabeled trees with 10 vertices. For an ordered pair of vertices $V, W \in G$, let $d(V, W)$ be the fewest number of edges needed to walk from vertex V to vertex W along G. Estimate the expected value of

$$\sum_{V,W \in G} d(V, W).$$

If your answer is A and the correct answer is E, you will receive $\lfloor \max\left(0, 20 - 2\lfloor |E - A| \rfloor^{0.5}\right) \rfloor$ points. *Note:* a *tree* is a connected graph with no cycles.



______ 35. **[30]** A positive integer is *beautiful* if it can be expressed as $a^k + b^k$ for positive integers a, b, and k with $k > 1$. A positive integer is a *peacock* if it is ten digits and uses every digit from 0 to 9 exactly once. Estimate the number of beautiful peacocks. If A is the correct answer and E is your estimate, you will receive $\left\lfloor 20 \min\left(\frac{A}{E}, \frac{E}{A}\right)^3 \right\rfloor$.

______ 36. **[30]** Let P_n denote the polynomial with integer coefficients such that

$$P_n(\cos\theta)\sin\theta = \sin((n+1)\theta)$$

for all real θ. Estimate

$$\sum_{i=1}^{\infty} \frac{P_i(2)}{P_{2i}(2)}.$$

If your answer is A and the correct answer is E, you will be awarded $\left\lfloor 20 \min\left(\frac{A}{E}, \frac{E}{A}\right)^2 \right\rfloor$ points.