Accuracy Round

LMT Spring 2025

May 3, 2025

1. **[6]** Find
$$\text{lcm}(2,025)+\text{lcm}(20,25)+\text{lcm}(202,5).$$

2. **[7]** Ben has 25 total pennies, nickles, and dimes. He has at least 14 pennies and nickles, at least 14 nickles and dimes, and at least 14 pennies and dimes. Find the maximum possible total value of Ben's coins, in cents.

3. **[7]** Equilateral triangle ABC and square $ADEF$ both have side length 2 and have a common vertex A. Find the maximum possible area of BCE.

4. **[8]** Find the number of ways to arrange the numbers $n^1, n^2, n^3, \dots, n^{2025}$, such that there exists a real number n for which the list is in strictly increasing order.

5. **[8]** At a party with 2025 families, one-third have exactly one child, one-third have exactly two children, and one-third have exactly three children. A child is selected uniformly at random from all children at the party. Find the expected number of siblings they have.

6. **[9]** A word is *admitting* if it shares more letters with the word "orz" than the word "zro". Find the number of admitting 3-letter words that don't contain any letters other than "o", "r", and "z". A letter is shared if both words have the same letter in the same position.

7. **[9]** Let $ABCD$ be a trapezoid with $AB \parallel CD$, $AB \perp BC$, and $\frac{CD}{AB}=2$. Let AC and BD intersect at X. Suppose triangle BCX has area 1. Find the area of pentagon $ABCDX$.

8. **[10]** Suppose N is a positive integer for which $5N^2$ has exactly 18 factors, and $7N^2$ has exactly 20 factors. Find the value of N.

9. **[11]** There are 16 distinct expressions that can be created by filling in each blank of
$$2 \_\_ 2 \_\_ 2 \_\_ 5 \_\_ 5$$
with a $\times$ or $\div$ symbol. Find the sum of all 16 expressions.

10. **[11]** Each cell in a 4×4 grid contains a light switch which is initially off. Jonathan can toggle all the switches along a diagonal of the grid. Find the number of possible on/off configurations he can achieve. (The diagonal does not have to be a main diagonal.)

11. **[12]** Let $ADMITS$ be a cyclic equiangular hexagon. Let U be the intersection of segments $\overline{MS}$ and $\overline{AI}$. Given that $\overline{IS}=14$ and $\overline{ST}=10$, find the sum of areas $[SAM]+[TSUI]$.

12. **[12]** Let a, b, c, and d be integers satisfying the equations
$$a^2+bd+c=0 \qquad b^2+ca+d=1 \qquad c^2+db+a=2 \qquad d^2+ac+b=3$$
Find all possible ordered tuples (a,b,c,d).

13. **[13]** Let p,q be distinct primes such that
$$k=\frac{p^2+15\cdot q^3}{p+15\cdot q}$$
is an integer. Find the sum of all possible k.

14. **[13]** An 8×8 checkerboard is colored black and white such that the top left cell is white. A set of 4 distinct cells of the checkerboard is called *stable* if

   - the four cells are not all the same color,
   - the centers of the four cells form a square with sides parallel to the gridlines, and
   - the top right cell of the four is black.

   Find the number of ways to select 16 stable sets such that no black cell is in two sets.

15. **[14]** Let $ABCDE$ be a convex pentagon such that $DE = EA = AB = BC$ and $DE \parallel CB$. Suppose $\angle EAB = 120^\circ$, $CD = 11$, and triangle ACD has area 27. Find $AD^2 + AC^2$.

16. **[TIEBREAKER]** In a 5×5 grid, each cell is colored either white or black. No 2×2 square contains all cells of the same color. Furthermore, all white cells are connected, i.e. one can walk from any white cell to any other white cell by only walking through edge-adjacent white cells. The same is true of black cells. Estimate the number of such grids, where rotations and reflections are considered distinct. If your answer is A and the true answer is E, your score on this tiebreaker will be $\min\left(\frac{E}{A}, \left(\frac{A}{E}\right)^{1.5}\right)$. A higher tiebreaker score is better.