Team Round

LMT Fall 2025

December 13th, 2025

1. **[20]** Let $ABCD$ be a square with side length 1. Circle ω is tangent to AB and AD, and passes through C. Find the area of ω.

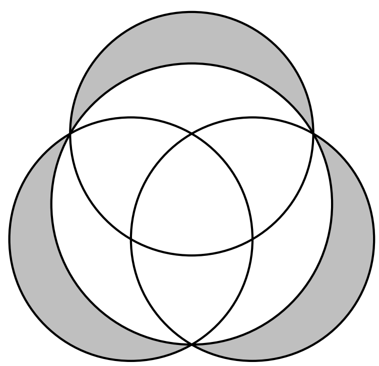
2. **[20]** Let x and y be positive integers satisfying $x^y = 2^{18}$. Find the smallest possible value of $x + y$.

3. **[20]** Find the number of ways to color a 3×3 grid with 3 colors such that each color is used exactly 3 times and no cells of the same color share an edge.

4. **[20]** Ben and Jerry are swimming across a really big pool. It takes 40 minutes for Ben to travel the length of the pool and 30 minutes for Jerry to travel the length of the pool. As soon as either of them reach one end of the pool, they turn around and start swimming back towards the other end. If Ben and Jerry simultaneously start swimming in the same direction from one end of the pool, find the number of minutes it takes before they meet again.

5. **[30]** Let f be a function satisfying $f(x) + f\left(1 - \frac{1}{x}\right) = x$. Find $f(2)$.

6. **[35]** Three circles with radius 2 are drawn such that their centers form an equilateral triangle with side length 2. These circles intersect with each other at exactly 6 points, 3 of which are the centers of the circles. Another circle is drawn passing through the other 3 intersections. Find the area of the shaded region.



7. **[40]** Eddie is at $(0,0)$ on the coordinate plane and wants to get to $(2,10)$. If he is at (x, y), he can move to one of $(x+2, y)$, $(x-2, y)$, $(x+1, y+1)$, $(x-1, y+1)$. He does not revisit any points along his path and his x and y coordinates are always greater than or equal to 0 and less than or equal to 2 and 10 respectively. Find the number of ways Eddie can get to $(2,10)$.

8. **[40]** ST, GT, and CT are 3 friends that are each given 3 unique positive integers from 1-9 inclusive such that none of them share any numbers. They then have the following perfectly logical conversation.

   - ST: My 3 numbers form a geometric sequence.
   - GT: ST has the largest number, and I don't know if CT's numbers form an arithmetic sequence.

   If GT's numbers are a, b, c with $a < b < c$, find $100a + 10b + c$.

9. **[45]** Given

$$(x-21)(x-22)(x-23)(x-24) = 2025,$$

   find the sum of all possible values of $(x-20)(x-25)$.

10. **[45]** A circular chip of radius $\frac{1}{2}$ is placed uniformly at random on an infinite grid of unit squares. Find the expected number of unit squares that the chip at least partially covers.

11. **[50]** Let $ABCD$ be a parallelogram and ω be a circle with radius 2. Suppose ω is tangent to AB, AD, BC at X, Y, C respectively and intersect CD at Z. If $AX = 1$, find DZ.

12. **[50]** Let f be a function such that $f(0) = 1$ and for positive integers x,

$$f(x) = \begin{cases} 0 & \text{if } x \equiv 0 \pmod 3 \\ \frac{1}{3}f\left(\frac{x-1}{3}\right) & \text{if } x \equiv 1 \pmod 3 \\ \frac{1}{9}f\left(\frac{x-2}{3}\right) & \text{if } x \equiv 2 \pmod 3 \end{cases}.$$

Find $\sum_{n=1}^{\infty} f(n)$.

13. **[55]** Let ℓ be a line. Pick points A,B on the same side of ℓ such that the distances to ℓ are 7,3 and $AB = 5$. Distinct points S, T are chosen on ℓ such that $(ASB), (ATB)$ are tangent to ℓ. Find ST.

14. **[60]** Let a_n be a sequence such that $a_1 = 1$ and

$$a_n = 1 + \frac{2025}{a_{n-1}} + \frac{2025}{a_{n-1}a_{n-2}} + \cdots + \frac{2025}{a_{n-1}a_{n-2}\cdots a_1}$$

Find $\lfloor a_{2025} \rfloor$.

15. **[70]** A bag has 5 red marbles, 3 green marbles, and 2 blue marbles. Marbles are drawn from the bag one at a time, with replacement. Find the expected number of draws that are required for each color to appear at least once, given that red appears first, then green, then blue.