# Theme Round

## LMT "Fall"

## December 17, 2022

## Tetris

Tetris is a Soviet block game developed in 1984, probably to torture misbehaving middle school children. Nowadays, Tetris is a game that people play for fun, and we even have a mini-event featuring it, but it shall be used on this test for its original purpose. The 7 Tetris pieces, which will be used in various problems in this theme, are as follows:



1. **[6]** Each piece has area 4. Find the sum of the perimeters of each of the 7 Tetris pieces.

2. **[8]** In a game of Tetris, Qinghan places 4 pieces every second during the first 2 minutes, and 2 pieces every second for the remainder of the game. By the end of the game, her average speed is 3.6 pieces per second. Find the duration of the game in seconds.

3. **[10]** Jeff takes all 7 different Tetris pieces and puts them next to each other to make a shape. Each piece has an area of 4. Find the least possible perimeter of such a shape.

4. **[12]** Qepsi is playing Tetris, but little does she know: the latest update has added realistic physics! She places two blocks, which form the shape below. Tetrominoes $ABCD$ and $EFGHIJ$ are both formed from 4 squares of side length 1. Given that $CE = CF$, the distance from point $I$ to the line $AD$ can be expressed as $\frac{A\sqrt{B}-C}{D}$. Find $1000000A + 10000B + 100C + D$.
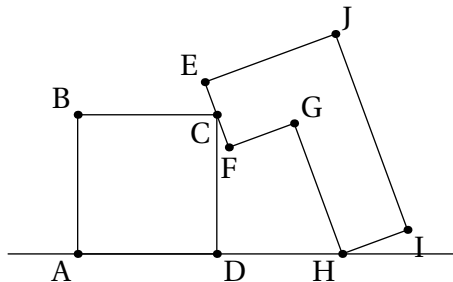


5. **[14]** Using the following tetrominoes:



Find the number of ways to tile the shape below, with rotation allowed, but reflection disallowed:
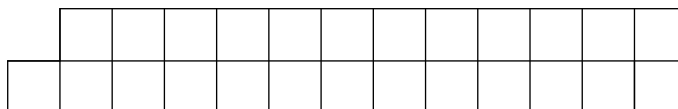
# World Cup

The World Cup, featuring 17 teams from Europe and South America, as well as 15 other teams that honestly don't have a chance, is a soccer tournament that is held once every four years. As we speak, Croatia and Morocco are locked in a battle that has no significance whatsoever on the winner, but if you would like live score updates nonetheless, feel free to ask your proctor, who has no obligation whatsoever to provide them.

1. **[6]** During the group stage of the World Cup, groups of 4 teams are formed. Every pair of teams in a group play each other once. Each team earns 3 points for each win and 1 point for each tie. Find the greatest possible sum of the points of each team in a group.

2. **[8]** In the semi-finals of the World Cup, the ref is bad and lets $11^2 = 121$ players per team go on the field at once. For a given team, one player is a goalie, and every other player is either a defender, midfielder, or forward. There is at least one player in each position. The product of the number of defenders, midfielders, and forwards is a mulitple of 121. Find the number of ordered triples (number of defenders, number of midfielders, number of forwards) that satisfy these conditions.

3. **[10]** Messi is playing in a game during the Round of 16. On rectangular soccer field $ABCD$ with $AB = 11$, $BC = 8$, points $E$ and $F$ are on segment $BC$ such that $BE = 3$, $EF = 2$, and $FC = 3$. If the distance between Messi and segment $EF$ is less than 6, he can score a goal. The area of the region on the field where Messi can score a goal is $a\pi + \sqrt{b} + c$, where $a$, $b$, and $c$ are integers. Find $10000a + 100b + c$.

4. **[12]** The workers are building the World Cup stadium for the 2022 World Cup in Qatar. It would take 1 worker working alone 4212 days to build the stadium. Before construction started, there were 256 workers. However, each day after construction, 7 workers disappear. Find the number of days it will take to finish building the stadium.

5. **[14]** In the penalty kick shootout, 2 teams each get 5 attempts to score. The teams alternate shots and the team that scores a greater number of times wins. At any point, if it's impossible for one team to win, even before both teams have taken all 5 shots, the shootout ends and no more shots are taken. If each team does take all 5 shots and afterwards the score is tied, the shootout enters sudden death, where teams alternate taking shots until one team has a higher score while both teams have taken the same number of shots.

   If each shot has a $\frac{1}{2}$ chance of scoring, the expected number of times that any team scores can be written as $\frac{A}{B}$ for relatively prime positive integers $A$ and $B$. Find $1000A + B$.

## Ephram

Ephram Chun is a senior and math captain at Lexington High School. He is well-loved by the freshmen, who seem to only listen to him. Other than being the father figure that the freshmen never had, Ephram is also part of the Science Bowl and Science Olympiad teams along with being part of the highest orchestra LHS has to offer. His many hobbies include playing soccer, volleyball, and the many forms of chess. We hope that he likes the questions that we've dedicated to him!

1. **[6]** Ephram is scared of freshmen boys. How many ways can Ephram and 4 distinguishable freshmen boys sit together in a row of 5 chairs if Ephram does not want to sit between 2 freshmen boys?

2. **[8]** Ephram, who is a chess enthusiast, is trading chess pieces on the black market. Pawns are worth \$100, knights are worth \$515, and bishops are worth \$396. Thirty-four minutes ago, Ephram made a fair trade: 5 knights, 3 bishops, and 9 rooks for 8 pawns, 2 rooks, and 11 bishops. Find the value of a rook, in dollars.

3. **[10]** Ephram is kicking a volleyball. The height of Ephram's kick, in feet, is determined by

$$h(t) = -\frac{p}{12} t^2 + \frac{p}{3} t,$$

   where $p$ is his kicking power and $t$ is the time in seconds. In order to reach the height of 8 feet between 1 and 2 seconds, Ephram's kicking power must be between reals $a$ and $b$. Find is $100a + b$.

4. **[12]** Disclaimer: No freshmen were harmed in the writing of this problem.

   Ephram has superhuman hearing: He can hear sounds up to 8 miles away. Ephram stands in the middle of a 8 mile by 24 mile rectangular grass field. A freshman falls from the sky above a point chosen uniformly and randomly on the grass field. The probability Ephram hears the freshman bounce off the ground is $P\%$. Find $P$ rounded to the nearest integer.
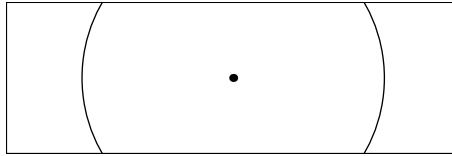


5. **[14]** Ephram and Brandon are playing a version of chess, sitting on opposite sides of a $6 \times 6$ board. Ephram has 6 white pawns on the row closest to himself, and Brandon has 6 black pawns on the row closest to himself. During each player's turn, their only legal move is to move one pawn one square forward towards the opposing player. Pawns cannot move onto a space occupied by another pawn. Players alternate turns, and Ephram goes first (of course). Players take turns until there are no more legal moves for the active player, at which point the game ends. Find the number of possible positions the game can end in.