Team Round

LMT "Fall"

December 17, 2022

1. **[9]** Let x be the positive integer satisfying $5^2 + 28^2 + 39^2 = 24^2 + 35^2 + x^2$. Find x.
2. **[10]** Ada rolls a standard 4-sided die 5 times. The probability that the die lands on at most two distinct sides can be written as $\frac{A}{B}$ for relatively prime positive integers A and B. Find $1000A + B$.
3. **[11]** Billiam is distributing his ample supply of balls among an ample supply of boxes. He distributes the balls as follows: he places a ball in the first empty box, and then for the greatest positive integer n such that all n boxes from box 1 to box n have at least one ball, he takes all of the balls in those n boxes and puts them into box $n+1$. He then repeats this process indefinitely. Find the number of repetitions of this process it takes for one box to have at least 2022 balls.
4. **[12]** Find the least positive integer ending in 7 with exactly 12 positive divisors.
5. **[13]** Let H be a regular hexagon with side length 1. The sum of the areas of all triangles whose vertices are all vertices of H can be expressed as $A\sqrt{B}$ for positive integers A and B such that B is square-free. What is $1000A + B$?
6. **[15]** An isosceles trapezoid $PQRS$, with $\overline{PQ} = \overline{QR} = \overline{RS}$ and $\angle PQR = 120°$, is inscribed in the graph of $y = x^2$ such that QR is parallel to the x-axis and R is in the first quadrant. The x-coordinate of point R can be written as $\frac{\sqrt{A}}{B}$ for positive integers A and B such that A is square-free. Find $1000A + B$.
7. **[17]** A regular hexagon is split into 6 congruent equilateral triangles by drawing in the 3 main diagonals. Each triangle is colored 1 of 4 distinct colors. Rotations and reflections of the figure are considered nondistinct. Find the number of possible distinct colorings.
8. **[19]** An odd positive integer n can be expressed as the sum of two or more consecutive integers in exactly 2023 ways. Find the greatest possible nonnegative integer k such that 3^k is a factor of the least possible value of n.
9. **[21]** In isosceles trapezoid $ABCD$ with $AB < CD$ and $BC = AD$, the angle bisectors of $\angle A$ and $\angle B$ intersect CD at E and F respectively, and intersect each other outside the trapezoid at G. Given that $AD = 8$, $EF = 3$, and $EG = 4$, the area of $ABCD$ can be expressed as $\frac{a\sqrt{b}}{c}$ for positive integers a, b, and c, with a and c relatively prime and b squarefree. Find $10000a + 100b + c$.
10. **[23]** Let $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$ and $\beta = \sin^{-1}\left(\frac{3}{5}\right)$.
$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{\cos(\alpha n + \beta m)}{2^n 3^m}.$$
can be written as $\frac{A}{B}$ for relatively prime positive integers A and B. Find $1000A + B$.