Theme Round

Lexington High School

December 7th, 2019

Joe Quigley

This section was written in memory of Joseph "Joe" John IV Quigley, who passed away last October. Joe Quigley ran the Math Club in Lexington for over twenty-four years, making math accessible and fun for students of all ages and abilities. His love of math was only eclipsed by his love for teaching, and he will be greatly missed by the entire Lexington community for his humor, patience, and dedication to his students.

1. Joe Quigley writes the following expression on the board for his students to evaluate:

$$2 \times 4 + 3 - 1$$

   However, his students have not learned their order of operations so they randomly choose which operations to perform first, second, and third. How many different results can the students obtain?

2. Joe Quigley flies airplanes around the Cartesian plane. There are two fuel stations, one at $(8, 12)$ and another at $(25, 5)$. He must station his home on the x-axis, and wants to be the same distance away from both station. Compute this distance.

3. Joe Quigley has 12 students in his math class. He will distribute N worksheets among the students. Find the smallest positive integer N for which any such distribution of the N worksheets among the 12 students results in at least one student having at least 3 worksheets.

4. Joe Quigley writes the number $4^{4^{4^4}}$ on the board. He correctly observes that this number has $2^a + b$ factors, where a and b are positive integers. Find the maximum possible value of a.

5. Joe Quigley is teaching his students geometric series and asks them to compute the value of the following series:

$$\sum_{x \geq 1} \frac{x(x+1)}{2^x}$$

   Compute this value.

Astronomy

6. Alex and Anka go to see stars. Alex counts thirteen fewer stars than Anka, and both of the numbers of stars they counted are square numbers. It is known that exactly eight stars were counted by both of them. How many distinct stars did they count in total?

7. Three planets with coplanar, circular, and concentric orbits are shown on the backside of this page. The radii of the three circles are 3, 4, and 5. Initially, the three planets are collinear. Every hour, the outermost planet moves one-sixth of its full orbit, the middle planet moves one-fourth of its full orbit, and the innermost planet moves one-third of its full orbit (A full orbit occurs when a planet returns to its initial position). Moreover, all three planets orbit in the same direction. After three hours, what is the area of the triangle formed by the planets as its three vertices?
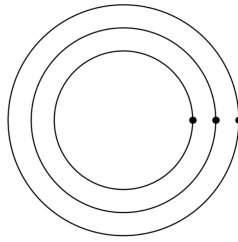
8. Planets X and Y are following circular orbits around the same star. It takes planet X 120 hours to complete a full trip around the star, and it takes planet Y 18 hours to complete a full trip around the star. If both planets begin at the same location in their orbit and travel clockwise, how many times will planet Y pass planet X in the time it takes planet X to complete a full trip around the star?

9. In a certain stellar system, four asteroids form a rectangle. Your spaceship lies in the rectangle formed. The distances between three of these asteroids and your space ship are 1 light minute, 8 light minutes, and 4 light minutes. If the distance between your space ship and the last asteroid is an integer number of light minutes away then how far away from the last asteroid is your space ship?

10. Two lost lovers, Laxe and Kaan, are both standing on the equators of planets with radius 13 miles. The center of the planets are 170 miles apart. At some time, both of them are as close to each other as possible. The planets rotate in opposite directions of each other at the same rate. What is the maximum possible distance between Laxe and Kaan such that they are still able to see each other?

Holidays

11. Festivus occurs every year on December 23rd. In 2019, Festivus will occur on a Monday. On what day will Festivus occur in the year 2029?

12. Leakey, Marpeh, Arik, and Yehau host a Secret Santa, where each one of them is assigned to give a present to somebody other than themselves. How many ways can the gifting be assigned such that everyone receives exactly one gift?

13. How many permutations of the word $CHRISTMAS$ are there such that the S's are not next to each other and there is not a vowel anywhere between the two S's?

14. Dasher, Dancer, Prancer, Vixen, Comet, Cupid, Donner, Blitzen, and Rudolph (9 reindeer) are guiding Santa's sleigh. They are arranged in a 3×3 array. You, the elf, have a big responsibility. You must place Santa's reindeer in a manner so that all of Santa's requests are met:

   - Donner is forgetful and must be put in the back row so Santa can keep an eye on Donner.
   - Additionally, Rudolph's big red nose distracts Donner, so Rudolph and Donner cannot be in the same column.
   - Finally, Comet is the fastest and must be put in the front row.

   How many options do you have for arranging Santa's reindeer?

15. Marpeh has a Christmas tree in the perfect shape of a right circular cone. The tree has base radius 8 inches and slant height 32 inches. He wants to place 3 ornaments on the surface of the tree with the following rules:

   - The red ornament is placed at the top of the tree.
   - The yellow ornament is placed along the circumference of the base of the tree.
   - The blue ornament is placed such that it is the same distance from the red and yellow ornaments when traveling on the surface of the tree.

   What is the furthest possible surface distance that the blue ornament could be from the red ornament?