Theme Round

Lexington High School

April 7, 2018

Olympics

1. The number of ways to rearrange PYEONGCHANG OLYMPIC can be expressed as $\frac{L!}{(M!)^T}$. Compute $L+M+T$.
2. How many ways are there to color each ring in the olympic logo with the colors blue, yellow, black, green or red such that no two rings that intersect are the same color?

3. At the Winter Olypmics, Belarus and the Czech Republic won B and C medals respectively. Altogether, they won 10 medals. When the number of medals the US won is divided by $B+C$ it leaves a remainder of 3 and when it is divided by BC it leaves a remainder of 2. What is the least possible number of medals the U.S. could have won?
4. Let l_1 denote the string CURLING. Let l_{k+1} be the sequence formed by inserting the substring 'CURLING' between each consecutive letter of l_k. For example,

$$l_2 = \mathbf{C}CURLING\mathbf{U}CURLING\mathbf{R}CURLING\mathbf{L}CURLING\mathbf{I}CURLING\mathbf{N}CURLING\mathbf{G}.$$

   What is the 1537th letter in l_{2018}?
5. Five spectators of an Olympic wrestling match each stand at a random point around the circumference of the circular ring. Find the probability that they are all contained within a 90 degree arc of the circle.

Trivia

6. Randy is playing a trivia game with 6 questions. Each question has 3 answer choices and if he answers all 6 questions correctly, he wins 5000 dollars. What is the expected amount of money Randy will win?

7. It has recently been proven that a sudoku puzzle requires at least 17 numbers to be uniquely solvable.

   We section a 4×4 grid of boxes into four 2×2 squares. In each square we place the number $1, 2, 3$, or 4. An arrangement is called *sudoku-like* if there is exactly one of $1, 2, 3, 4$ in each row, column, and 2×2 box. How many sudoku-like arrangements are there? An example is given below.

1	2	3	4
4	3	1	2
2	1	4	3
3	4	2	1

8. A *superperfect* number is a number n such that if $\sigma(n)$ denotes the sum of its factors, then $\sigma(\sigma(n)) = 2n$. For example, $\sigma(16) = 1 + 2 + 4 + 8 + 16 = 31$ and $\sigma(31) = 32$.

   A and B are distinct numbers such that they each have exactly 10 factors and $|A - B| = 1$. Find the minimum possible value of $A + B$.

9. A trivia game awards up to 2018 dollars split evenly among all of its winners such that each winner gets the maximum possible integer number of dollars. In a particular game if one more person had won each winner would have gotten two fewer dollars. How many possible number of winners are there for this game?

10. The number 0 didn't exist until 628 AD, when it was introduced by the Indian mathematician Brahmagupta. The concept of 0 did exist much before then.

    Let $\oplus$ be a binary operator such that for any 3 real numbers a, b, c, we have

    $$(a \oplus b) \oplus c = a \oplus (b \oplus c)$$

    and

    $$a \oplus b \oplus c = 4abc - 2ab - 2bc - 2ac + a + b + c.$$

    Find all possible values of $20 \oplus 18$.

Star Wars

11. R2-D2 is trying to break into a room. He realizes that the code is the same as $A+B$, where A and B are two positive integers greater than 1 such that

$$20_A + 12_B = 19_{B+A}$$

and

$$20_A - 12_B = 11_{B-A},$$

where the A subscript means the number is in base A. What is the password?

12. Tatooine is located at the point $(0,4)$, Hoth is located at the point $(3,5)$, Alderaan was located at the point $(2,1)$ and Endor is located at $(0,0)$. At what point should the Death Star stop in order to minimize its combined distance to each location?

13. How many ways can Chancellor Palpatine place indistinguishable galactic credits on a 4 by 8 grid such that each row has exactly 3 credits and each column has exactly 6 credits?

14. Rey and Kylo Ren are each flying through space. Rey starts at $(-2,2)$ and ends at $(2,2)$ and Kylo Ren starts at $(-1,1)$ and ends at $(1,1)$. They can only travel 1 unit in the x or y direction at a time. Additionally, each of them must go to the x-axis before going to their respective destinations and take the shortest path possible to do so. Finally, their paths never intersect at any point. How many distinct configurations of paths can they take?

15. A triangle ABC is formed with A is the current location of Darth Vader's spaceship, B is the location of the Rebel Base, and C is the location of the Death Star. Let D, E, and F be the locations of the spaceships of Luke Skywalker, Han Solo, and Princess Leia, respectively. It is true that D is the foot of the A-altitude, E is the foot of the A-angle bisector, and F is the foot of the A-median. Suppose the 4 segments on BC (some possibly of length 0), when measured in light-years, form an arithmetic sequence (in any order). What is the largest possible value of $\frac{BC}{AB}$?