Team Round

LMT Spring 2023

May 20, 2023

1. **[10]** Given the following system of equations:

$$\begin{cases} RI+G+SP &= 50 \\ RI+T+M &= 63 \\ G+T+SP &= 25 \\ SP+M &= 13 \\ M+RI &= 48 \\ N &= 1 \end{cases}$$

   Find the value of L that makes $LMT + SPRING = 2023$ true.

2. **[11]** How many integers of the form n^{2023-n} are perfect squares, where n is a positive integer between 1 and 2023 inclusive?

3. **[12]** Beter Pai wants to tell you his fastest 40-line clear time in Tetris, but since he does not want Qep to realize she is better at Tetris than he is, he does not tell you the time directly. Instead, he gives you the following requirements, given that the correct time is t seconds:

   - $t < 100$.
   - t is prime.
   - $t-1$ has 5 proper factors.
   - all prime factors of $t+1$ are single digits.
   - $t-2$ is a multiple of 3.
   - $t+2$ has 2 factors.

   Find t.

4. **[13]** There exists a certain right triangle with the smallest area in the 2D coordinate plane such that all of its vertices have integer coordinates but none of its sides are parallel to the x- or y-axis. Additionally, all of its sides have distinct, integer lengths. What is the area of this triangle?

5. **[14]** How many ways are there to place the integers from 1 to 8 on the vertices of a regular octagon such that the sum of the numbers on any 4 vertices forming a rectangle is even? Rotations and reflections of the same arrangement are considered distinct.

6. **[16]** Find the least positive integer m such that $105 \mid 9^{(p^2)} - 29^p + m$ for all prime numbers $p > 3$.

7. **[18]** Jerry writes down all binary strings of length 10 without any two consecutive 1s. How many 1s does Jerry write?

8. **[20]** Let x, y, and z be positive reals that satisfy the system

$$\begin{cases} x^2+xy+y^2 &= 10 \\ x^2+xz+z^2 &= 20 \\ y^2+yz+z^2 &= 30 \end{cases}$$

Find $xy + yz + xz$.

9. **[22]** In $\triangle ABC$, $AB = 13$, $BC = 14$, and $CA = 15$. Let E and F be the feet of the altitudes from B onto CA, and C onto AB, respectively. A line ℓ is parallel to EF and tangent to the circumcircle of ABC on minor arc BC. Let X and Y be the intersections of ℓ with AB and AC respectively. Find XY.

10. **[24]** The sequence $a_0, a_1, a_2, \ldots$ is defined such that $a_0 = 2 + \sqrt{3}$, $a_1 = \sqrt{5 - 2\sqrt{5}}$, and

$$a_n a_{n-1} a_{n-2} - a_n + a_{n-1} + a_{n-2} = 0.$$

Find the least positive integer n such that $a_n = 1$.