Speed Round

LMT Spring 2023

May 20, 2023

1. **[6]** Evaluate $(2-0)^2 \cdot 3 + \frac{20}{2+3}$.

2. **[6]** Let $x = 11 \cdot 99$ and $y = 9 \cdot 101$. Find the sum of the digits of $x \cdot y$.

3. **[6]** A rectangle is cut into two pieces. The ratio between the areas of the two pieces is $3:1$ and the positive difference between those areas is 20. What's the area of the rectangle?

4. **[6]** Edgeworth is scared of elevators. He is currently on floor 50 of a building, and he wants to go down to floor 1. Edgeworth can go down at most 4 floors each time he uses the elevator. What's the minimum number of times he needs to use the elevator to get to floor 1?

5. **[6]** There are 20 people at a party. Fifteen of those people are normal and 5 are crazy. A normal person will shake hands once with every other normal person, while a crazy person will shake hands twice with every other crazy person. How many total handshakes occur at the party?

6. **[6]** Wam and Sang are chewing gum. Gum comes in packages, each package consisting of 14 sticks of gum. Wam eats 6 packs and 9 individual sticks of gum. Sang wants to eat twice as much gum as Wam. How many packs of gum must Sang buy?

7. **[6]** At Lakeside Health School (LHS), 40% of students are male and 60% of the students are female. If half of the students at the school take biology, and the same number of male and female students take biology, to the nearest percent, what percent of female students take biology?

8. **[6]** Evin is bringing diluted raspberry iced tea to the annual Lexington Math Team party. He has a cup with 10 mL of iced tea and a 2000 mL cup of water with 10% raspberry iced tea. If he fills up the cup with 20 more mL of 10% raspberry iced tea water, what percent of the solution will be iced tea?

9. **[6]** Tree 1 starts at height 220m and grows continuously at 3m per year. Tree 2 starts at height 20m and grows at 5m during the first year, 7m per during the second year, 9m during the third year, and in general $(3+2n)$m in the nth year. After which year is Tree 2 taller than Tree 1?

10. **[6]** Leo and Chris are playing a game in which Chris flips a coin. The coin lands on heads with probability $\frac{499}{999}$, tails with probability $\frac{499}{999}$, and it lands on its side with probability $\frac{1}{999}$. For each flip of the coin, Leo agrees to give Chris 4 dollars if it lands on heads, nothing if it lands on tails, and 2 dollars if it lands on its side. What's the expected value of the number of dollars Chris gets after flipping the coin 17 times?

11. **[6]** Ephram has a pile of balls, which he tries to divide into piles. If he divides the balls into piles of 7, there are 5 balls that don't get divided into any pile. If he divides the balls into piles of 11, there are 9 balls that aren't in any pile. If he divides the balls into piles of 13, there are 11 balls that aren't in any pile. What is the minimum number of balls Ephram has?

12. **[6]** Let $\triangle ABC$ be a triangle such that $AB = 3$, $BC = 4$, and $CA = 5$. Let F be the midpoint of AB. Let E be the point on AC such that $EF \parallel BC$. Let CF and BE intersect at D. Find AD.

13. **[6]** Compute the sum of all even positive integers $n \le 1000$ such that:

$$\operatorname{lcm}(1,2,3,\dots(n-1)) \neq \operatorname{lcm}(1,2,3,\dots n).$$

14. **[6]** Find the sum of all palindromes with 6 digits in binary, including those written with leading zeroes.

15. **[6]** What is the side length of the smallest square that can entirely contain 3 non-overlapping unit circles?

16. **[6]** Find the sum of the digits in the base 7 representation of 6250000. Express your answer in base 10.

17. **[6]** A number n is called *sus* if n^4 is one more than a multiple of 59. Compute the largest sus number less than 2023.

18. **[6]** Michael chooses real numbers a and b independently and randomly from $(0,1)$. Given that a and b differ by at most $\frac{1}{4}$, what is the probability a and b are both greater than $\frac{1}{2}$?

19. **[6]** In quadrilateral $ABCD$, $AB = 7$ and $DA = 5$, $BC = CD$, $\angle BAD = 135^\circ$ and $\angle BCD = 45^\circ$. Find the area of $ABCD$.

20. **[6]** Find the value of
$$\sum_{i|210} \sum_{j|i} \left\lfloor \frac{i+1}{j} \right\rfloor$$

21. **[6]** Let a_n be the number of words of length n with letters $\{A,B,C,D\}$ that contain an odd number of As. Evaluate a_6.

22. **[6]** Detective Hooa is investigating a case where a criminal stole someone's pizza. There are 69 people involved in the case, among whom one is the criminal and another is a witness of the crime. Every day, Hooa is allowed to invite any of the people involved in the case to his rather large house for questioning. If on some given day, the witness is present and the criminal is not, the witness will reveal who the criminal is. What is the minimum number of days of questioning required such that Hooa is guaranteed to learn who the criminal is?

23. **[6]** Find
$$\sum_{n=2}^{\infty} \frac{2n+10}{n^3+4n^2+n-6}.$$

24. **[6]** Let $\triangle ABC$ be a triangle with circumcircle ω such that $AB = 1$, $\angle B = 75^\circ$, and $BC = \sqrt{2}$. Let lines ℓ_1 and ℓ_2 be tangent to ω at A and C respectively. Let D be the intersection of ℓ_1 and ℓ_2. Find $\angle ABD$ (in degrees).

25. **[6]** Find the sum of the prime factors of $14^6 + 27$.