LMT Spring 2023 Guts Round - Part 1   Team Name: _____

_____ 1. **[4]** Solve the maze

_____ 2. **[4]** Billiam can write a problem in 30 minutes, Jerry can write a problem in 10 minutes, and Evin can write a problem in 20 minutes. Billiam begins writing problems alone at 3:00 PM until Jerry joins him at 4:00 PM, and Evin joins both of them at 4:30 PM. Given that they write problems until the end of math team at 5:00 PM, how many full problems have they written in total?

_____ 3. **[4]** How many pairs of positive integers (n, k) are there such that $\binom{n}{k} = 6$?

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LMT Spring 2023 Guts Round - Part 2   Team Name: _____

_____ 4. **[5]** Find the sum of all integers $b > 1$ such that the expression of 143 in base b has an even number of digits and all digits are the same.

_____ 5. **[5]** Ini thinks that $a\#b = a^2 - b$ and $a\&b = b^2 - a$, while Mimi thinks that $a\#b = b^2 - a$ and $a\&b = a^2 - b$. Both Ini and Mimi try to evaluate $6\&(3\#4)$, each using what they think the operations & and # mean. What is the positive difference between their answers?

_____ 6. **[5]** A unit square sheet of paper lies on an infinite grid of unit squares. What is the maximum number of grid squares that the sheet of paper can partially cover at once? A grid square is partially covered if the area of the grid square under the sheet of paper is nonzero - i.e., lying on the edge only does not count.

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_____ 7. **[6]** Maya wants to buy lots of burgers. A burger without toppings costs \$4, and every added topping increases the price by 50 cents. There are 5 different toppings for Maya to choose from, and she can put any combination of toppings on each burger. How much would it cost for Maya to buy 1 burger for each distinct set of toppings? Assume that the order in which the toppings are stacked onto the burger does not matter.

_____ 8. **[6]** Consider square $ABCD$ and right triangle PQR in the plane. Given that both shapes have area 1, $PQ = QR$, $PA = RB$, and P, A, B and R are collinear, find the area of the region inside both square $ABCD$ and $\triangle PQR$, given that it is not 0.

_____ 9. **[6]** Find the sum of all n such that n is a 3-digit perfect square that has the same tens digit as $\sqrt{n}$, but that has a different ones digit than $\sqrt{n}$.

_____ 10. **[7]** Jeremy writes the string:

$$LMTLMTLMTLMTLMTLMT$$

on a whiteboard ("LMT" written 6 times). Find the number of ways to underline 3 letters such that from left to right the underlined letters spell LMT.

_____ 11. **[7]** Compute the remainder when 12^{2022} is divided by 1331.

_____ 12. **[7]** What is the greatest integer that cannot be expressed as the sum of 5s, 23s, and 29s?

_____ 13. **[9]** Square $ABCD$ has point E on side BC, and point F on side CD, such that $\angle EAF = 45°$. Let $BE = 3$, and $DF = 4$. Find the length of FE.

_____ 14. **[9]** Find the sum of all positive integers k such that

1. k is the power of some prime.
2. k can be written as 5654_b for some $b > 6$.

_____ 15. **[9]** If $\sqrt[3]{x} + \sqrt[3]{y} = 2$ and $x + y = 20$, compute $\max(x, y)$

LMT Spring 2023 Guts Round - Part 6   Team Name: _____

_____ 16. **[11]** Triangle ABC with $AB < AC$ is inscribed in a circle. Point D lies on the circle and point E lies on side AC such that $ABDE$ is a rhombus. Given that $CD = 4$ and $CE = 3$, compute AD^2.

_____ 17. **[11]** Wam and Sang are walking on the coordinate plane. Both start at the origin. Sang walks to the right at a constant rate of 1 m/s. At any positive time t (in seconds), Wam walks with a speed of 1 m/s with a direction of t radians clockwise of the positive x-axis. Evaluate the square of the distance between Wam and Sang in meters after exactly 5π seconds.

_____ 18. **[11]** Mawile is playing a game against Salamance. Every turn, Mawile chooses one of two moves: Sucker Punch or Iron Head, and Salamance chooses one of two moves: Dragon Dance or Earthquake. Mawile wins if the moves used are Sucker Punch and Earthquake, or Iron Head and Dragon Dance. Salamance wins if the moves used are Iron Head and Earthquake. If the moves used are Sucker Punch and Dragon Dance, nothing happens and a new turn begins. Mawile can only use Sucker Punch up to 8 times. All other moves can be used indefinitely. Assuming both Mawile and Salamance play optimally, find the probability that Mawile wins.

LMT Spring 2023 Guts Round - Part 7   Team Name: _____

_____ 19. **[13]** Ephram is attempting to organize what rounds everyone is doing for the NEAML competition. There are 4 rounds, of which everyone must attend exactly 2. Additionally, of the 6 people on the team (Ephram, Wam, Billiam, Hacooba, Matata, and Derke), exactly 3 must attend every round. In how many different ways can Ephram organize the teams like this?

_____ 20. **[13]** For some 4th degree polynomial $f(x)$, the following is true:

- $f(-1) = 1$.
- $f(0) = 2$.
- $f(1) = 4$.
- $f(-2) = f(2) = f(3)$.

Find $f(4)$.

_____ 21. **[13]** Find the minimum value of the expression $\sqrt{5x^2 - 16x + 16} + \sqrt{5x^2 - 18x + 29}$ over all real x.

_____ 22. **[15]** Let O and I be the circumcenter and incenter, respectively, of $\triangle ABC$ with $AB = 15$, $BC = 17$, and $CA = 16$. Let $X \neq A$ be the intersection of line AI and the circumcircle of $\triangle ABC$. Find the area of $\triangle IOX$.

_____ 23. **[15]** Find the sum of all integers x such that there exist integers y and z such that

$$x^2 + y^2 = 3(2016^z) + 77.$$

_____ 24. **[15]** Evaluate

$$\left\lfloor \sum_{i=1}^{2022} \frac{1}{\sqrt{i}} \right\rfloor = \left\lfloor \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{2022}} \right\rfloor.$$

_____ 25. **[10]** Either:

1. Submit -2 as your answer and you'll be rewarded with two points OR

2. Estimate the number of teams that choose the first option. If your answer is within 1 of the correct answer, you'll be rewarded with three points, and if you are correct, you'll receive ten points

_____ 26. **[10]** Jeff is playing a turn-based game that starts with a positive integer n.

Each turn, if the current number is n, Jeff must choose one of the following:

1. The number becomes the nearest perfect square to n

2. The number becomes $n - a$, where a is the largest digit in n

Let $S(k)$ be the least number of turns Jeff needs to get from the starting number k to 0. Estimate

$$\sum_{k=1}^{2023} S(k).$$

If your estimation is E and the actual answer is A, you will receive $\max\left(\left\lfloor 10 - \left|\frac{E-A}{6000}\right|\right\rfloor, 0\right)$ points.

_____ 27. **[10]** Estimate the smallest positive integer n such that if N is the area of the n-sided regular polygon with circumradius 100, $10000\pi - N < 1$ is true.

If your estimation is E and the actual answer is A, you will receive $\max\left(\left\lfloor 10 - \left|10 \cdot \log_3\left(\frac{E}{A}\right)\right|\right\rfloor, 0\right)$ points.