Accuracy Round

LMT Spring 2023

May 20, 2023

1. **[6]** Andrew writes down all of the prime numbers less than 50. How many times does he write the digit 2?

2. **[8]** Evaluate $2023^2 - 2022^2 + 2021^2 - 2020^2$.

3. **[8]** Phoenix is counting positive integers starting from 1. When he counts a perfect square greater than 1, he restarts at 1, skipping that square the next time. For example, the first 10 numbers Phoenix counts are $1, 2, 3, 4, 1, 2, 3, 5, 6, 7\ldots$. How many numbers will Phoenix have counted after counting 100 for the first time?

4. **[10]** Rectangle $ABCD$ has side lengths $AB = 3$ and $BC = 7$. Let E be a point on BC, and let F be the intersection of DE and AC. Given that $[CDF] = 4$, find $\frac{DF}{FE}$.

5. **[10]** Let
$$N = \sum_{i=0}^{512} i \binom{512}{i}.$$
What is the greatest integer a such that 2^a is a divisor of N?

6. **[10]** Aidan, Boyan, Calvin, Derek, Ephram, and Fanalex are all chamber musicians at a concert. In each performance, any combination of musicians of them can perform for all the others to watch. What is the minimum number of performances necessary to ensure that each musician watches every other musician play?

7. **[12]** In $\triangle ABC$, $AB = 13$, $BC = 14$, and $CA = 15$. Let D be a point on BC such that $BD = 6$. Let E be a point on CA such that $CE = 6$. Finally, let F be a point on AB such that $AF = 6$. Find the area of $\triangle DEF$.

8. **[12]** Ephram is taking his final exams. He has 7 exams and his school holds finals over 3 days. For a certain arrangement of finals, let f be the maximum number of finals Ephram takes on any given day. Find the expected value of f.

9. **[12]** Evin's calculator is broken and can only perform 3 operations: Operation 1: Given a number x, output $2x$. Operation 2: Given a number x, output $4x + 1$. Operation 3: Given a number x, output $8x + 3$. After initially given the number 0, how many numbers at most 128 can he make?

10. **[12]** Positive integers a, b, and c satisfy $a^2 + b^2 = c^3 - 1$ where $c \le 40$. Find the sum of all distinct possible values of c.

11. **[TIEBREAKER]** Estimate the value of
$$\sum_{n=1}^{2023} \left(1 + \frac{1}{n}\right)^n$$
to 3 decimal places.