LMT Fall 2023 Guts Round Solutions- Part 1

Team Name:

1. **[12]** Calculate

$$(4!-5!+2^5+2^6)\cdot\frac{12!}{7!}+(1-3)(4!-2^4).$$

Proposed by Evin Liang

Solution. $\boxed{-16}$

As $4!-5!+2^5+2^6=0$, the answer is $(1-3)(4!-2^4)=-2\cdot 8=\boxed{-16}$. □

2. **[12]** The expression $\sqrt{9!+10!+11!}$ can be expressed as $a\sqrt{b}$ for positive integers a and b, where b is squarefree. Find a.

Proposed by Jacob Xu

Solution. $\boxed{792}$

We rewrite the expression as $\sqrt{9!\cdot(1+10+10\cdot 11)}=\sqrt{9!\cdot 121}$. Now this is easy to simplify as $11\sqrt{9!}=11\sqrt{2^7\cdot 3^4\cdot 5\cdot 7}=11\cdot 2^3\cdot 3^2\sqrt{2\cdot 5\cdot 7}=792\sqrt{70}$. So $a=792$. □

3. **[12]** For real numbers a and b, $f(x)=ax^{10}-bx^4+6x+10$ for all real x. Given that $f(42)=11$, find $f(-42)$.

Proposed by Ethan Xu

Solution. $\boxed{-493}$

$f(-x)=a(-x)^{10}-b(-x)^4-6x+10$

$f(-x)=ax^{10}-bx^4-6x+10$

$f(-x)=(ax^{10}-bx^4+6x+10)-12x$

$f(-x)=f(x)-12x$

$f(-42)=f(42)-(12)(42)=11-504=\boxed{\text{-493}}$ □

LMT Fall 2023 Guts Round Solutions- Part 2

Team Name:

4. **[15]** How many positive integers less than or equal to 2023 are divisible by 20, 23, or both?

Proposed by Evin Liang

Solution. $\boxed{184}$

The answer is $\left\lfloor\frac{2023}{20}\right\rfloor+\left\lfloor\frac{2023}{23}\right\rfloor-\left\lfloor\frac{2023}{460}\right\rfloor=101+87-4=184$. □

5. **[15]** Larry the ant crawls along the surface of a cylinder with height 48 and base radius $\frac{14}{\pi}$. He starts at point A and crawls to point B, traveling the shortest distance possible. What is the maximum this distance could be?

Proposed by Jonathan Liu

Solution. $\boxed{48+\dfrac{28}{\pi}}$

2 cases: 1st case is only crawling on the rounded surface, which can be unfolded into a right triangle with base 14 height 48, which is length 50 2nd case is when the two points are the two centers of the bases, which has length $\frac{14}{\pi}+48+\frac{14}{\pi}=48+\frac{28}{\pi}$, which is about 56.9, which is larger than 50 so the answer is $\boxed{48+\dfrac{28}{\pi}}$ □

______________ 6. **[15]** For a given positive integer n, Ben knows that $\lfloor 20x\rfloor = n$, where x is real. With that information, Ben determines that there are 3 distinct possible values for $\lfloor 23x\rfloor$. Find the least possible value of n.

Proposed by Muztaba Syed

Solution. $\boxed{6}$

We have $\frac{n}{20}\le x<\frac{n+1}{20}$. This means that $\frac{23n}{20}\le 23x<\frac{23n+23}{20}$. For there to be 3 values there need to be 2 integers between $\frac{23n}{20}$ and $\frac{23n+23}{20}$. This means there are 2 multiples of 20 between $23n$ and $23n+23$. This means that $23n$ is slightly less (namely $18,19 \pmod{20}$) than a multiple of 20, and by inspection we see $n=\boxed{6}$ works. □

LMT Fall 2023 Guts Round Solutions- Part 3

Team Name:

______________ 7. **[18]** Let ABC be a triangle with area 1. Points D, E, and F lie in the interior of $\triangle ABC$ in such a way that D is the midpoint of AE, E is the midpoint of BF, and F is the midpoint of CD. Compute the area of DEF.

Proposed by Evin Liang

Solution. $\boxed{\dfrac{1}{7}}$

Using some equal bases you can get that $[DEF]=[BDE]=[BDA]$, and likewise these are all equal to $[BEC]=[EFC]=[ADF]=[AFC]$. Thus the area of $\triangle ABC$ is 7 times the area of $\triangle DEF$, so the answer is $\boxed{\dfrac{1}{7}}$. □

______________ 8. **[18]** Edwin and Amelia decide to settle an argument by running a race against each other. The starting line is at a given vertex of a regular octahedron and the finish line is at the opposite vertex. Edwin has the ability to run straight through the octahedron, while Amelia must stay on the surface of the octahedron. Given that they tie, what is the ratio of Edwin's speed to Amelia's speed?

Proposed by Edwin Zhao

Solution. $\boxed{\dfrac{\sqrt{6}}{3}}$

WLOG suppose the vertices of the octahedron are at permutations of $(\pm1,0,0)$. Eddie goes from $(1,0,0)$ to $(-1,0,0)$, for a total distance of 2. Amelia goes from $(1,0,0)$ to $\left(0,\frac{1}{2},\frac{1}{2}\right)$ to $(-1,0,0)$, for a total distance of $\sqrt{6}$. Thus, if they tie, Eddie runs $\frac{2}{\sqrt{6}}=\boxed{\dfrac{\sqrt{6}}{3}}$ times the speed of Amelia. □

______________ 9. **[18]** Jxu is rolling a fair three-sided die with faces labeled 0, 1, and 2. He keeps going until he rolls a 1, immediately followed by a 2. What is the expected number of rolls Jxu makes?

Proposed by Evin Liang

Solution. $\boxed{9}$

Let E be the answer and F be the expected number of rolls when the last roll is 1. Then we have $E = 1 + \frac{2}{3}E + \frac{1}{3}F$ and $F = 1 + \frac{1}{3}E + \frac{1}{3}F$. The solution is $(E, F) = (9, 6)$, so the answer is $\boxed{9}$. □

LMT Fall 2023 Guts Round Solutions- Part 4 Team Name: ______________

______ 10. **[21]** For real numbers x and y, $x + xy = 10$ and $y + xy = 6$. Find the sum of all possible values of $\frac{x}{y}$.

Proposed by Evin Liang

Solution. $\boxed{\frac{16}{3}}$

Subtract to get $x - y = 4$, so $x = 4 + y$, $y^2 + 5y - 6 = 0$, and so we have y is 1 or -6, and in these cases x is 5 or -2. So the answer is $\frac{5}{1} + \frac{-2}{-6} = \boxed{\frac{16}{3}}$. □

______ 11. **[21]** Derek is thinking of an odd two-digit integer n. He tells Aidan that n is a perfect power and the product of the digits of n is also a perfect power. Find the sum of all possible values of n.

Proposed by Aidan Duncan

Solution. $\boxed{130}$

Some guess and check gives $n = 49, 81 \implies \boxed{130}$ □

______ 12. **[21]** Let a three-digit positive integer $N = \overline{abc}$ (in base 10) be *stretchable* with respect to m if N is divisible by m, and when N's middle digit is duplicated an arbitrary number of times, it's still divisible by m. How many three-digit positive integers are *stretchable* with respect to 11? (For example, 432 is *stretchable* with respect to 6 because $433\dots32$ is divisible by 6 for any positive integer number of 3s.)

Proposed by Samuel Wang

Solution. $\boxed{8}$

Let the integer be abc (obviously base 10). abc and $abbc$ must thus be multiples of 11. However, this means that $abbc - abc = 900a + 100b$ is a multiple of 11, thus $9a + b$ is a multiple of 11 (or, $b \equiv 2a \mod 11$). We thus list out the possibilities for a, b: $a = 1 : b = 2$. This gives $c = 1$, or 121 $a = 2 : b = 4$. This gives $c = 2$, or 242 $a = 3, b = 6$. This gives $c = 3$, or 363 $a = 4, b = 8$. This gives $c = 4$, or 484 $a = 5, b = 10$. Obviously, $b \neq 10$. $a = 6, b = 1$. This gives $c = 6$, or 616 $a = 7, b = 3$. This gives $c = 7$, or 737 $a = 8, b = 5$. This gives $c = 8$, or 858 $a = 9, b = 7$. This gives $c = 9$, or 979 $a = 10, b = 9$. Obviously, $a \neq 10$. The total number is thus $\boxed{8}$. □

LMT Fall 2023 Guts Round Solutions- Part 5 Team Name: ______________

______ 13. **[27]** How many trailing zeroes are in the base-2023 expansion of 2023!?

Proposed by Evin Liang

Solution. $\boxed{63}$

We have $2023 = 7 \cdot 17^2$, so the number of trailing zeroes is $\min\left(v_7(2023!), \frac{v_{17}(2023!)}{2}\right)$. Clearly $\frac{v_{17}(2023!)}{2}$ is smaller, and we have $v_{17}(2023!) = \left\lfloor \frac{2023}{17} \right\rfloor + \left\lfloor \frac{2023}{17^2} \right\rfloor = 126$. So the answer is $\frac{126}{2} = \boxed{63}$. □

14. **[27]** The three-digit positive integer $k = \overline{abc}$ (in base 10, with a nonzero) satisfies

$$\overline{abc} = c^{2ab-1}.$$

Find the sum of all possible k.

Proposed by Muztaba Syed

Solution. $\boxed{853}$

$2ab-1$ has to be odd, so it really doesn't have many options, since it can't be greater than 9 and is at least 3. If it is 9 the only possibility is 512 which works. If $2ab-1=7$ the only possible 7th power is 128 which doesn't work. For $2ab-1=5$ the only possibility is $3^5 = 243$ which doesn't work. Finally checking $2ab-1=3$ we get 125, 216, 343, 512, 729, of which only 125 and 216 work. Thus the answer is $512+125+216 = \boxed{853}$. □

15. **[27]** For any positive integer k, let a_k be defined as the greatest nonnegative real number such that in an infinite grid of unit squares, no circle with radius less than or equal to a_k can partially cover at least k distinct unit squares. (A circle partially covers a unit square only if their intersection has positive area.)

Find the sum of all positive integers $n \le 12$ such that $a_n \ne a_{n+1}$.

Proposed by Peter Bai

Solution. $\boxed{38}$

Solution:

$a_1 : 0$ $a_2 : 0$ $a_3 : 0$ $a_4 : 0$ $a_5 : \frac{1}{2}$ $a_6 : \frac{1}{2}$ $a_7 : \frac{5}{8}$ $a_8 : \frac{\sqrt{2}}{2}$ $a_9 : \frac{\sqrt{2}}{2}$ $a_{10} : 1$ $a_{11} : 1$ $a_{12} : 1$

a_{13} is definitely bigger than 1

$4+6+7+9+12 = 38$ □

LMT Fall 2023 Guts Round Solutions- Part 6

Team Name:

16. **[33]** Let $p(x)$ and $q(x)$ be polynomials with integer coefficients satisfying $p(1) = q(1)$. Find the greatest integer n such that $\frac{p(2023)-q(2023)}{n}$ is an integer no matter what $p(x)$ and $q(x)$ are.

Proposed by Samuel Wang

Solution. $\boxed{2022}$

Solution: As $p(1) = q(1)$, p and q have the same sum of coefficients. Thus, we see that $p(2023) \equiv q(2023) \bmod 2022$. We also note that 2022 is the max, as we set $p(x) = 2$, $q(x) = x+1$, giving $p(2023) - q(2023) = 2022$, thus we cannot go higher, giving $n = \boxed{2022}$ □

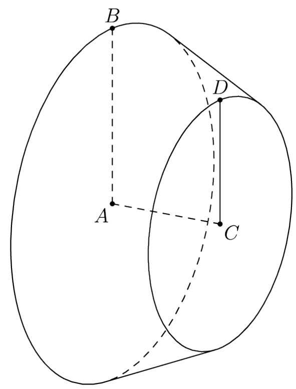
17. **[33]** Find all ordered pairs of integers (m, n) that satisfy $n^3 + m^3 + 231 = n^2m^2 + nm$.

Proposed by Evin Liang

Solution. $\boxed{(4,5) and (5,4)}$

This equation is $(n^2 - m)(m^2 - n) = 231$. The prime factorization of 231 is $2 \cdot 7 \cdot 11$ and the only solutions are $(n, m) = (4,5)$ and $(5,4)$. □

18. **[33]** Ben rolls the frustum-shaped piece of candy (shown below) in such a way that the lateral area is always in contact with the table. He rolls the candy until it returns to its original position and orientation.



Given that $AB = 4$ and $BD = CD = 3$, find the length of the path traced by A.

Proposed by Jerry Xu

Solution. $\boxed{\dfrac{64\pi}{3}}$

Key observation: The path traced out by the frustum is identical to the path traced out by it's cone. Denote the vertex of the cone as F, and the point opposite B on the base of the cone is E, the point opposite D on its base of the frustum as H, and the altitude from B to the ground as G. Consider the cross-section containing EBF. We have that $BE = 2AB = 8$, $DH = 2CD = 6$, and $BD = 3$. Since $\overline{BE} \parallel \overline{DH}$, we get that $\triangle BEF \sim \triangle DHF \implies BF = EF = 12$ (remember that $\triangle BEF$ is isosceles since its the cross-section of a cone). By dropping an altitude from F to BE we get that $[BEF] = 32\sqrt{2}$. Thus, since BG is an altitude, $BG = \dfrac{16\sqrt{2}}{3}$. By similar triangles, $AI = BG/2 = \dfrac{8\sqrt{2}}{3}$, and by pythag on AEI (noting that $AE = BE/2 = 4$) we have that $EI = \dfrac{4}{3}$ so $IF = \dfrac{32}{3}$. A is at a constant height and distance from F, so the answer is simply $2\pi \cdot \dfrac{32}{3} = \boxed{\dfrac{64\pi}{3}}$. □

LMT Fall 2023 Guts Round Solutions- Part 7

Team Name: ____________________________

______________ 19. **[39]** In their science class, Adam, Chris, Eddie and Sam are independently and randomly assigned an integer grade between 70 and 79 inclusive. Given that they each have a distinct grade, what is the expected value of the maximum grade among their four grades?

Proposed by Samuel Tsui

Solution. $\boxed{77.8}$

The different possible values of the maximum are $73, 74, \ldots, 79$. By stars and bars the number of ways it can be 73 is $\binom{3}{3}$, the number of ways it can be 74 is $\binom{4}{3}$ and so on up to the number of ways it can be 79 is $\binom{9}{3}$. Since there are a total of $\binom{10}{4}$ ways the expected value is $\frac{73\cdot\binom{3}{3}+74\cdot\binom{4}{3}+\cdots+79\cdot\binom{9}{3}}{\binom{10}{4}}$. Simplifying gives $\frac{72\left(\binom{3}{3}+\binom{4}{3}+\cdots+\binom{9}{3}\right)+\binom{3}{3}+2\binom{4}{3}+\cdots+7\binom{9}{3}}{\binom{10}{4}}$ which is equal to $\frac{72\cdot\binom{10}{4}+1218}{\binom{10}{4}}$ from hockey stick and a bit of calculation. Thus the answer is $72 + \frac{1218}{210} = 77\frac{4}{5}$. □

______________ 20. **[39]** Let $ABCD$ be a regular tetrahedron with side length 2. Let point E be the foot of the perpendicular from D to the plane containing $\triangle ABC$. There exist two distinct spheres ω_1 and ω_2, centered at points O_1 and O_2 respectively, such that both O_1 and O_2 lie on $\overrightarrow{DE}$ and both spheres are tangent to all four of the planes ABC, BCD, CDA, and DAB. Find the sum of the volumes of ω_1 and ω_2.

Proposed by Peter Bai

Solution. $\boxed{\frac{\sqrt{6}}{3}\pi}$

First of all, we notice that since everything is symmetrical about ray $\overrightarrow{DE}$, the points of tangency of ω_1 and ω_2 on plane ABC must both be the centroid of triangle ABC. Also due to a symmetry argument, the centroid of triangle ABC must also be point E. (This is possible to verify via repeated applications of the Pythagorean Theorem on various lengths in tetrahedron $ABCD$, but symmetry can be very helpful under time pressure.)

Let's first set up some conventions. Let r_1 and r_2 denote the radii of ω_1 and ω_2, respectively, and let point M be the midpoint of segment $\overline{AB}$.

First of all, drawing a rough diagram out on paper, we can see that ω_1 is actually just inscribed within tetrahedron ABC. ω_2 is a bit more difficult to think about, however, but it is also tangent to plane ABC at the same point, except on the other side.

We will begin by finding r_1. Consider plane ABC.

Since $\triangle ABC$ is equilateral, $\triangle EMB$ is a 30-60-90 triangle. As a result, we have $\overline{EM} = \frac{\sqrt{3}}{3}$. Additionally, since E is the centroid of $\triangle ABC$, we have $\overline{CM} = 3\overline{EM} = \sqrt{3}$.

While we already have E as one point of tangency, analogous 2-dimensional problems involving incircles of triangles usually require the consideration of 2 points of tangency. As a result, it makes sense to also look at the point of tangency between ω_1 and plane ABD. Let us denote this point as G, and again by a symmetry argument, G must be the centroid of $\triangle ABD$. We can now take a cross-section across plane CDM!

Note that $\overline{GO_1} = \overline{O_1E} = r_1$ and that ω_1 never actually contacts line $\overline{DC}$. Additionally, the diagram is symmetric about ray $\overrightarrow{MO_1}$, and thus $\overline{DG} = \overline{EC} = \frac{2\sqrt{3}}{3}$. Finally, by the Pythagorean Theorem, $\overline{DE} = \sqrt{(\overline{DM})^2 - (\overline{ME})^2} = \sqrt{\left(\sqrt{3}\right)^2 - \left(\frac{\sqrt{3}}{3}\right)^2} = \frac{2\sqrt{6}}{3}$.

We can see that $\triangle DGO_1$ is similar to $\triangle DEM$. As a result, we have $\frac{\overline{DG}}{\overline{DE}} = \frac{\overline{GO_1}}{\overline{ME}}$. Solving,

$$\frac{\overline{DG}}{\overline{DE}} = \frac{\overline{GO_1}}{\overline{EM}} \Rightarrow \frac{\frac{2\sqrt{3}}{3}}{\frac{2\sqrt{6}}{3}} = \frac{r_1}{\frac{\sqrt{3}}{3}} \Rightarrow \frac{1}{\sqrt{2}} = \frac{r_1}{\frac{\sqrt{3}}{3}} \Rightarrow r_1 = \frac{\sqrt{6}}{6}$$

Moving on to r_2, we can take a similar approach by letting the point of tangency between ω_2 and plane ABD be denoted as H. Taking the cross-section across plane CDM (or equivalently, DME), we begin by noting that $\overline{HO_2} = \overline{O_2E} = r_2$ and $\overline{HM} = \overline{ME} = \frac{\sqrt{3}}{3}$, giving us $\overline{DH} = \sqrt{3} + \frac{\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$. Additionally, $\triangle DME$ is similar to $\triangle DO_2H$. As a result, we have $\frac{\overline{DE}}{\overline{DH}} = \frac{\overline{ME}}{\overline{O_2H}}$. Solving,

$$\frac{\overline{DE}}{\overline{DH}} = \frac{\overline{ME}}{\overline{O_2H}} \Rightarrow \frac{\frac{2\sqrt{6}}{3}}{\frac{4\sqrt{3}}{3}} = \frac{\frac{\sqrt{3}}{3}}{r_2} \Rightarrow \frac{\sqrt{2}}{2} = \frac{\frac{\sqrt{3}}{3}}{r_2} \Rightarrow r_2 = \frac{2}{\sqrt{2}} * \frac{\sqrt{3}}{3} = \frac{\sqrt{6}}{3}$$

Finally, all we have to do now is calculate the sum of the volumes.

$$\frac{4}{3}\pi\left(\frac{\sqrt{6}}{6}\right)^3 + \frac{4}{3}\pi\left(\frac{\sqrt{6}}{3}\right)^3$$

$$= \frac{4}{3}\pi\left(\frac{\sqrt{6}}{6}\right)^3(1+2^3)$$

$$= \frac{4}{3}\pi * \frac{\sqrt{6}}{36} * 9$$

$$= \frac{4}{3}\pi * \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{6}}{3}\pi$$

$6+3=9$, which is our answer. □

______ 21. **[39]** Evaluate

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{(i+j+k+1)2^{i+j+k+1}}.$$

Proposed by Peter Bai

Solution. $\boxed{\frac{3}{2}}$

We begin with a substitution of $h = i + j + k$, allowing us to collapse the 3 nested summations into a single one in h. Of course, we have to account for that fact that almost all values of h are summed more than once (for example, $5 = 1+2+2 = 2+1+2$). Using stars and bars, we can see that there are $\binom{h+2}{2}$ possible values of (i, j, k) summing to any valid h.

As a result, our new summation is:

$$\begin{aligned}\sum_{h=0}^{\infty} \frac{1}{(h+1)2^{h+1}} \cdot \frac{(h+2)(h+1)}{2} &= \sum_{h=0}^{\infty} \frac{1}{2^{h+1}} \cdot \frac{h+2}{2} \\ &= \sum_{h=0}^{\infty} \frac{h+2}{2^{h+2}} \\ &= \sum_{h=2}^{\infty} \frac{h}{2^h}\end{aligned}$$

This is a simple arithmetico-geometric series, which finally evaluates to our answer of $\frac{3}{2}$. □

LMT Fall 2023 Guts Round Solutions- Part 8

Team Name: ____________________

__________ 22. **[45]** In $\triangle ABC$, let I_A, I_B, and I_C denote the A, B, and C-excenters, respectively. Given that $AB = 15$, $BC = 14$ and $CA = 13$, find $\frac{[I_A I_B I_C]}{[ABC]}$.

Proposed by Jerry Xu

Solution. $\boxed{81}$

By Herons, we have that

$$\begin{aligned}[ABC] &= \sqrt{21(21-13)(21-14)(21-15)} \\ &= \sqrt{21 \cdot 8 \cdot 7 \cdot 6} \\ &= \sqrt{2^4 \cdot 3^2 \cdot 7^2} \\ &= 84.\end{aligned}$$

Now, the exradius (r_A) is equivalent to

$$\frac{[ABC]}{s-a}, (^*)$$

where $s = \frac{a+b+c}{2}$ is the semiperimeter. We thus have that $r_A = 12, r_B = 10.5$ and $r_C = 14$. Note that

$$\begin{aligned}[I_A I_B I_C] &= [ABC] + [I_A BC] + [I_B AC] + [I_C AB] \\ &= 84 + \frac{10.5 \cdot 13}{2} + \frac{12 \cdot 14}{2} + \frac{14 \cdot 15}{2} \\ &= 341.25.\end{aligned}$$

(Note that the radius from I_A to BC is tangent and therefore forms a right angle, so that radius is also an altitude. The same applies for I_B to AC and I_C to AB.)

We thus get an answer of $\frac{341.25}{84} = \frac{65}{16} \longrightarrow \boxed{81}$. □

__________ 23. **[45]** The polynomial

$$x + 2x^2 + 3x^3 + 4x^4 + 5x^5 + 6x^6 + 5x^7 + 4x^8 + 3x^9 + 2x^{10} + x^{11}$$

has distinct complex roots $z_1, z_2, \ldots, z_n$. Find

$$\sum_{k=1}^{n} \left|\Re(z_n^2)\right| + \left|\Im(z_n^2)\right|,$$

where $\Re z$ and $\Im z$ indicate the real and imaginary parts of z, respectively. Express your answer in simplest radical form.

Proposed by Jerry Xu

Solution. $\boxed{3+2\sqrt{3}}$

Observe that $P(x) = x(1+x+x^2+x^3+x^4+x^5)^2$. Hence the roots are 0 and the 6th roots of unity excluding 1. 0 doesn't contribute to the sum. Among the other solutions, -1 squares to 1 and the rest square to $-\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$. Hence the answer is $1+4\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right) = \boxed{3+2\sqrt{3}}$. □

______ 24. **[45]** Given that $\sin 33° + 2\sin 161° \cdot \sin 38° = \sin n°$, compute the least positive integer value of n.

Proposed by Evin Liang

Solution. $\boxed{71}$

By double angle $\sin 38° = 2\sin 19° \cos 19°$. Additionally $\sin 161° = \sin 19°$ so $2\sin 161° \sin 38° = 4\sin^2 19° \cos 19°$. By triple angle $\cos 57° = \cos 19° - 4\sin^2 19° \cos 19°$, and therefore $\sin 33° + 2\sin 161° \sin 38° = \cos 19° = \sin 71°$. □

LMT Fall 2023 Guts Round Solutions- Part 9

Team Name: ______

______ 25. **[30]** Submit a prime between 2 and 2023, inclusive. If you don't, or if you submit the same number as another team's submission, you will receive 0 points. Otherwise, your score will be

$$\min(30, \lfloor 4 \cdot \ln(x) \rfloor),$$

where x is the positive difference between your submission and the closest valid submission made by another team.

Proposed by Edwin Zhao

Solution. $\boxed{TBD}$

Game Theory □

______ 26. **[30]** Sam, Derek, Jacob, and Muztaba are eating a very large pizza with 2023 slices. Due to dietary preferences, Sam will only eat an even number of slices, Derek will only eat a multiple of 3 slices, Jacob will only eat a multiple of 5 slices, and Muztaba will only eat a multiple of 7 slices. How many ways are there for Sam, Derek, Jacob, and Muztaba to eat the pizza, given that all slices are identical and order of slices eaten is irrelevant? If your answer is A and the correct answer is C, the number of points you receive will be:

$$\max\left(0, \left\lfloor 30\left(1-2\sqrt{\frac{|A-C|}{C}}\right)\right\rfloor\right)$$

Proposed by Sam Wang

Solution. $\boxed{6653921}$

Python program □

____________ 27. **[30]** Let $\Omega(k)$ denote the number of perfect square divisors of k. Compute

$$\sum_{k=1}^{10000} \Omega(k).$$

If your answer is A and the correct answer is C, the number of points you recieve will be

$$\max\left(0, \left\lfloor 30\left(1-4\sqrt{\frac{|A-C|}{C}}\right)\right\rfloor\right)$$

Proposed by Muztaba Syed

Solution. $\boxed{16307}$

Python Program.

Approximation technique: We can instead count the number of multiples of each perfect square less than 10000. This gives us

$$\sum_{k=1}^{100} \left\lfloor \frac{10000}{k^2} \right\rfloor$$

This is easier to approximate using the fact that $\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\cdots=\frac{\pi^2}{6}$. We can easily approximate $10000 \cdot \frac{\pi^2}{6} \approx 16450$. This will be too big because of the floors and because we don't go to infinity. Submitting 16450 would get 18 points already and this can be improved by approximating our overcount. $\square$