6th Annual Lexington Math Tournament Individual Round

March 28, 2015

Name _____ Team _____

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1. What is $\sqrt[2015]{2^0 1^5}$?
2. What is the ratio of the area of square $ABCD$ to the area of square $ACEF$?
3. 2015 in binary is 11111011111, which is a palindrome. What is the last year which also had this property?
4. What is the next number in the following geometric series: 1020100, 10303010, 104060401?
5. A circle has radius A and area r. If $A = r^2\pi$, then what is the diameter, C, of the circle?
6. If
$$O + N + E = 1$$
$$T + H + R + E + E = 3$$
$$N + I + N + E = 9$$
$$T + E + N = 10$$
$$T + H + I + R + T + E + E + N = 13$$
Then what is the value of O?
7. By shifting the initial digit, which is 6, of the positive integer N to the end (for example, 65 becomes 56), we obtain a number equal to $\frac{N}{4}$. What is the smallest such N?
8. What is $\sqrt[3]{\frac{2015!(2013!)+2014!(2012!)}{2013!(2012!)}}$
9. How many permutations of the digits of 1234 are divisible by 11?
10. If you choose 4 cards from a normal 52 card deck (with replacement), what is the probability that you will get exactly one of each suit (there are 4 suits)?
11. If LMT is an equilateral triangle, and MATH is a square, such that point A is in the triangle, then what is HL/AL?
12. If
$$\begin{array}{rr} & \texttt{LHS} \\ + & \texttt{HIGH} \\ + & \texttt{SCHOOL} \\ = & \texttt{SOCOOL} \end{array}$$
and $\{M, A, T, H, S, L, O, G, I, C\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, then what is the ordered pair $(M + A + T + H, [T + e + A + M])$ where e is $2.718\ldots$ and $[n]$ is the greatest integer less than or equal to n?
13. There are 5 marbles in a bag. One is red, one is blue, one is green, one is yellow, and the last is white. There are 4 people who take turns reaching into the bag and drawing out a marble without replacement. If the marble they draw out is green, they get to draw another marble out of the bag. What is the probability that the 3rd person to draw a marble gets the white marble?

14. Let a "palindromic product" be a product of numbers which is written the same when written back to front, including the multiplication signs. For example, $234 * 545 * 432$, $2 * 2 * 2 * 2$, and $14 * 41$ are palindromic products whereas $2 * 14 * 4 * 12$, $567 * 567$, and $2 * 2 * 3 * 3 * 2$ are not. 2015 can be written as a "palindromic product" in two ways, namely $13 * 5 * 31$ and $31 * 5 * 13$. How many ways can you write 2016 as a palindromic product without using 1 as a factor?

15. Let a sequence be defined as $S_n = S_{n-1} + 2S_{n-2}$, and $S_1 = 3$ and $S_2 = 4$. What is $\sum_{n=1}^{\infty} \frac{S_n}{3^n}$?

16. Put the numbers 0-9 in some order so that every 2-digit substring creates a number which is either a multiple of 7, or a power of 2.

17. Evaluate $\cfrac{8+\cfrac{8+\cfrac{8+\cfrac{8+\cfrac{8+\cfrac{8+\cfrac{8+\cfrac{8+\cfrac{8+\cfrac{8+\cdots}{3+\cdots}}{3+\cdots}}{3+\cdots}}{3+\cdots}}{3+\cdots}}{3+\cdots}}{3+\cdots}}{3+\cdots}}{3+\cfrac{8+\cfrac{8+\cfrac{8+\cfrac{8+\cfrac{8+\cfrac{8+\cfrac{8+\cfrac{8+\cfrac{8+\cdots}{3+\cdots}}{3+\cdots}}{3+\cdots}}{3+\cdots}}{3+\cdots}}{3+\cdots}}{3+\cdots}}{3+\cdots}}}$, assuming that it is a positive real number.

18. 4 non-overlapping triangles, each of area A, are placed in a unit circle. What is the maximum value of A?

19. What is the sum of the reciprocals of all the (positive integer) factors of 120 (including 1 and 120 itself)

20. How many ways can you choose 3 distinct elements of $\{1, 2, 3, ...4000\}$ to make an increasing arithmetic series?