5th Annual Lexington Math Tournament Individual Round

April 12, 2014

Name ______________________ Team ______________________

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1. What is $6 \times 7 + 4 \times 7 + 6 \times 3 + 4 \times 3$?

2. How many integers n have exactly $\sqrt{n}$ factors?

3. A triangle has distinct angles 3x+10, 2x+20, and x+30. What is the value of x?

4. If 4 people of the Math Club are randomly chosen to be captains, and Henry is one of the 30 people eligible to be chosen, what is the probability that he is not chosen to be captain?

5. a, b, c, d is an arithmetic sequence with difference x such that a, c, d is a geometric sequence. If b is 12, what is x? (Note: the difference of an aritmetic sequence can be positive or negative, but not 0)

6. What is the smallest positive integer that contains only 0s and 5s that is a multiple of 24

7. If ABC is a triangle with side lengths 13,14, and 15, what is the area of the triangle made by connecting the points at the midpoints of its sides?

8. How many ways are there to order the numbers 1,2,3,4,5,6,7,8 such that 1 and 8 are not adjacent?

9. Find all ordered triples of nonnegative integers (x, y, z) such that x + y + z = xyz.

10. Noah inscribes equilateral triangle ABC with area $\sqrt{3}$ in a cricle. If BR is a diameter of the circle, then what is the arc length of Noah's ARC?

11. Today, 4/12/14, is a palindromic date, because the number without slashes 41214 is a palindrome. What is the last palindromic date before the year 3000?

12. Every other vertex of a regular hexagon is connected to form an equilateral triangle. What is the ratio of the area of the triangle to that of the hexagon?

13. How many ways are there to pick four cards from a deck, none of which are the same suit or number as another, if order is not important?

14. Find all functions f from $\mathbb{R} \to \mathbb{R}$ such that $f(x+y) + f(x-y) = x^2 + y^2$.

15. What are the last four digits of $1(1!) + 2(2!) + 3(3!) + ... + 2013(2013!)$

16. In how many distinct ways can a regular octagon be divided up into 6 non-overlapping triangles?

17. Find the sum of the solutions to the equation $\frac{1}{x-3} + \frac{1}{x-5} + \frac{1}{x-7} + \frac{1}{x-9} = 2014$

18. How many integers n have the property that (n+1)(n+2)(n+3)(n+4) is a perfect square of an integer?

19. A quadrilateral is inscribed in a unit circle, and another one is circumscribed. What is the minimum possible area in between the two quadrilaterals?

20. In blindfolded solitary tic-tac-toe, a player starts with a blank 3-by-3 tic-tac-toe board. On each turn, he randomly places an "X" in one of the open spaces on the board. The game ends when the player gets 3 Xs in a row, in a column, or in a diagonal as per normal tic-tac-toe rules. (Note that only Xs are used, not Os). What fraction of games will run the maximum 7 amount of moves?