2nd Annual Lexington Mathematical Tournament
Theme Round

April 2, 2011

1 Interdisciplinary

1. Knowing very little about history, Bill assumes that the Hundred Years' War lasted exactly 100 years. Given that the war actually lasted for 116 years (from 1337 to 1453), what is the absolute value of the percent error of Bill's assumption? Express your answer as a decimal to the nearest tenth.

2. The Epic Cycle of Ancient Greek poems includes five poems, the *Cypria*, the *Aethiopis*, the *Iliupersis*, the *Nosti*, and the *Telegony*. Each of these poems consists of some number of books of verse. The average number of books making up the last four poems is 3.5. When the *Cypria* is included among them, the average number of books among the five poems is 5. How many books of verse make up the *Cypria*?

3. Zaroug is taking a test and must select the four noble gases from the set { Helium, Argon, Boricon, Neon, Radium, Xenon } and then arrange them in order of increasing mass. However, he only knows that there are exactly four noble gases in the set and that all six elements have different masses. Thus, he picks four elements from the set at random and then arranges them randomly. What is the probability he picks the four noble gases and orders them correctly?

4. Let $ABCD$ be a rectangle with $AB = 4$ and $BC = 12$. A ball is fired from vertex A towards side $\overline{BC}$ at 10 units per second and bounces off a point E on $\overline{BC}$ such that $BE/BC = 1/4$. Every time the ball bounces off a wall, it loses energy in such a way that its speed is halved instantly. In addition, the angle at which the ball comes in is equal to the angle at which the ball bounces off the wall. Given that the ball only loses speed at the bounce points, how many seconds does it take the ball to reach vertex D?

5. When two resistors in a circuit are connected in parallel, one of resistance m and one of resistance n, the equivalent resistance of the circuit R is given by $\frac{1}{R} = \frac{1}{m} + \frac{1}{n}$. How many ordered pairs (m, n) of positive integers are such that the equivalent resistance of a circuit with two parallel resistors of resistances m and n is equal to 10?

2 Schafkopf

Schafkopf is a five-player game that has its roots in Germany - it is now popular (sometimes under the name Sheepshead) in locales such as Wisconsin, Canada/USA Mathcamp, and economics classes at Lexington High School. It is played with a standard deck of playing cards, with all cards between 2 and 6, inclusive, removed - there are thus four each of 7, 8, 9, 10, Jack, Queen, King, and Ace. However, the suits differ from the standard - all queens, jacks, and diamonds form the "trump" suit, and the 7, 8, 9, 10, King, and Ace in clubs, hearts, and spades form the three other suits.

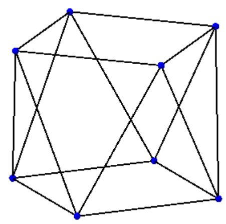
6. A card is drawn at random from a standard deck of 52 cards. What is the probability that it is used in the game of Schafkopf?

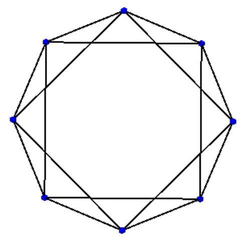
7. A card is drawn at random from a Schafkopf deck. What is the probability that it is part of the "trump" suit?

8. In Schafkopf, 6 cards are dealt to each player, with the two left over forming the "blind". What is the probability that both cards in the blind are queens?

9. Carl, J., Leon, Peter, and Ryan sit down at a circular table to play Schafkopf. If Peter does not want to sit next to J., in how many ways can the five of them sit down? Assume that each seat is numbered 1 through 5, and any two configurations in which someone is sitting in a different seat in the two configurations are distinct.

10. Let the points S, C, H, A, F, K, O, P be the vertices of a cube of side length 2 such that S is directly above F, C is directly above K, H is directly above O, and A is directly above P. Square $SCHA$ is rotated 45 degrees about its center and in its plane to square $S'C'H'A'$. The vertices of square $S'C'H'A'$ are then each connected to the two closest vertices of square $FKOP$, creating a new polyhedron $S'C'H'A'FKOP$ whose side and top views are shown on the left and right, respectively. Find the volume of this polyhedron.





3 Sophomore Eating Habits

Note: Strictly speaking, none of the questions actually feature sophomore eating habits.

11. Darwin is buying pies, which are flat and circular, to throw at people on April Fools'. He can either buy one large pie with radius 4 or a whole number of small pies with radius 2. How many small pies does Darwin need to buy such that the total area of the small pies is greater than one large pie?

12. Hao is selling cookies at a 2 day math competition. On the first day, he sold some number of cookies and some number of ramen packs, and earned \$14. The next day, he sold twice as many cookies and three times as many ramen packs, and earned \$37. If cookies are worth \$1 each and ramen packs are worth \$0.50 each, how many ramen packs did Hao sell on the first day?

13. Surya is going to serve curry turkey to his friends such that if he has a spherical turkey of radius x, he must cut it into the whole number of pieces numerically closest to $x/2011$. If Surya needs to serve 2011 guests with no extra pieces left over and so that each person gets exactly one piece, how many different possible integer values are there for the radius of the spherical turkey?

    Note: At the competition, the wording stated that "everyone gets a piece"

14. In Flatland, Peijin is packing a circular pizza of area 45π into a rhombus $ABCD$ such that the pizza is tangent to all four sides of the rhombus. It is given that $\angle A$ is obtuse and that the distance between the two tangency points closest to A is 6. Find the perimeter of the rhombus pizza box in simplest radical form.

15. Malik is adding 2011 flower decorations to a circular cake for a wedding along 2011 spaces planned out around the circumference of the cake, numbered 1 through 2011 counterclockwise. He places the first flower in space 1 and then cycles n spaces counterclockwise to a new space. If the space is empty, he places a flower in it, and doesn't otherwise. He then continues to cycle n spaces counterclockwise regardless of whether he placed a flower or not. For example, if $n = 7$, then after placing the first flower, he moves to space $1 + 7 = 8$, places a flower, moves to space $8 + 7 = 15$, places a flower, and so on. For what value of n, between 1 and 2011 inclusive, will the third to last flower Malik places end up in space 6?